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The power of swarming in personal clouds under bandwidth budget

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ABSTRACT

Users are unceasingly relying on personal clouds (like Dropbox, Box, etc) to store, edit and retrieve their files stored in remote servers. These systems generally follow a client–server model to distribute the files to end-users. This means that they require a huge amount of bandwidth to meet the requirements of their clients. Personal clouds with limited bandwidth budget can benefit from the upload speed of the clients interested in the same content to improve the quality of service. This can be done by introducing a peer-to-peer protocol, BitTorrent for instance, when the load on a certain content becomes high. The main challenge is to decide when to switch to BitTorrent and how to allocate the cloud's available bandwidth to the different clients. In this paper, we propose an algorithm for the allocation of the cloud's bandwidth. Based on the current load and the predefined quality of service constraints, the algorithm identifies the most suitable protocol for each swarm and provides the corresponding bandwidth allocation. We validate the algorithm using a real trace of the Ubuntu One system and the results show important gains in the download times experienced by the clients.

1. Introduction

Nowadays, users are unceasingly relying on cloud storage services (like Dropbox, Google Drive or Box, etc) to store, edit and retrieve their data stored in remote servers and which can be accessed all over the Internet. Such systems are hosted by cloud-based datacenters spread all over the world and are generally equipped with a set of features that allow sharing and collaboration between the users. That is why these popular applications account for a major share of Internet traffic today (Drago, 2013).

Small and medium-sized personal clouds with limited budget constraints generally have fixed amount of bandwidth. This bandwidth is shared by all the concurrent active end-users, which might jeopardize the overall quality of service especially when the demand becomes high. As a matter of fact, these systems are based on a client–server architecture and the default content distribution protocol is usually HTTP or HTTPS. This means that all download requests are handled by a central entity which sends the requested content in a single stream. Unfortunately, such transfer is limited by the narrowest network condition along the way, or by the server being overloaded by requests from many clients.

To cope with these limitations, the cloud can benefit from the clients' upload capacities to overcome its bandwidth limits. This can be done by using the BitTorrent protocol (Cohen, 2003) to distribute the files that are shared between a set of devices. In such scenarios, it is possible to benefit from the common interest of users in the same file and use their own upload bandwidth to offload the cloud from doing all the serving. However, the use of BitTorrent may incur a longer download time compared to HTTP especially for small files (Chaabouni et al., 2014). The main challenge is to decide for each swarm which approach is more suitable for transferring the requested files (client-server or peer-assisted), and how much cloud bandwidth should be allocated to each swarm.

In this paper, we study the relationship between the cloud bandwidth allocated to a swarm of clients and the resulting download time for the end-users. We also propose a bandwidth allocation and protocol decision algorithm that evaluates for each swarm the most suitable protocol and returns the amount of bandwidth to be allocated, based on the current load on the cloud. Our key contributions are as follows:

- We analyze the relationship between the amount of cloud bandwidth allocated to a given swarm and the resulting download time. Based on a fixed quality of service constraint, we calculate the amount of seed bandwidth needed to ensure a given ratio between the download times in HTTP and BitTorrent.

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- We propose a dynamic algorithm which uses simple parameters that can be collected by the system and evaluates the efficacy of using HTTP and BitTorrent as a distribution protocol for each requested file. Based on the load on the cloud and the predefined switching constraints, the algorithms decides the most suitable protocol for each case and provides the corresponding bandwidth allocations at the swarm level. This algorithm can be applied in cloud-based content distribution systems to achieve important improvements in the overall quality of service.
- We evaluate the efficiency of our proposal using two simulators. The first simulates the default behavior of the cloud where each download operation is treated individually and the content is delivered using HTTP. The second simulator simulates the bandwidth distribution and switching algorithm where BitTorrent can be used along with HTTP to distribute content. We validate both approaches using a real trace of the Ubuntu One[1] system: we vary the switching constrains and the cloud upload speed limits and measure the degree of improvement in download time of the involved clients using our algorithm (BitTorrent and HTTP together) compared to the use of HTTP alone. The results show important improvements in the download time experienced by the peers.

The remainder of this paper is organized as follows: first, we discuss related work in Section 2 and present some background on BitTorrent and personal clouds in Section 3. Second, we present the architecture of our personal cloud system and highlights the main differences compared to the classic personal clouds in Section 4. In Section 5, we state the bandwidth allocation problem and propose an algorithm to solve it. The algorithm is evaluated later in Section 6 using a trace of a real personal cloud system. Finally, Section 7 concludes the paper.

2. Related work

Various related works focus on how to efficiently distribute content to a set of users, from classical content distribution networks (CDNs) to online, multicast streaming of live content. Peer-to-peer is considered one of the technologies that has proven its efficiency in reducing the load on the servers and improving the distribution time of the clients (Karagiannis et al., 2005). BitTorrent (Cohen, 2003) is one of the peer-to-peer protocols that has attracted the researchers' attention and several studies were dedicated to the analysis, modeling and measurements of the BitTorrent ecosystem (Qiu and Srikant, 2004; Izal et al., 2004; Pouwelse et al., 2005; Piatek et al., 2007).

BitTorrent was also introduced in cloud environments and many previous works have focused on reducing download times for large contents using BitTorrent in cloud settings. BitTorrent has proven its efficiency not only for bulk synchronous content distribution (Sweha et al., 2011), but also for reducing transfer times for cloud virtual images (Wartel et al., 2010; Schmidt et al., 2010; Reich et al., 2010). For example, in Reich et al. (2010), authors demonstrate that their BitTorrent-based solution for distributing virtual machines delivers up to an $30\times$ speedup over traditional remote file system approaches.

In this paper, we tackle the problem of the efficient distribution of the files to end-user in personal cloud systems. These systems have become very popular lately and there have been important works related to the benchmark and design of these services (Drago et al., 2012, 2013; Gracia-Tinedo et al., 2013; Garcia-Lopez et al., 2014; Tinedo et al., 2015). To the best of our knowledge, we were the first to propose to use both HTTP and BitTorrent together in cloud systems (Chaabouni et al., 2013, 2014).

In Chaabouni et al. (2013), we introduced the idea of transparent switching from HTTP to BitTorrent upon detection of a certain critical mass demand on a specific content. The threshold was placed on the number of users requesting the same files. The system tested with each new request whether the current number of requesters of the corresponding file passed the predefined threshold or not. When the threshold was reached, the system decided to adopt BitTorrent instead of HTTP in order to avoid bottlenecks on the one hand, and to save cloud bandwidth on the other hand.

In Chaabouni et al. (2014), we investigated further the threshold at which the system should switch from HTTP to BitTorrent. Instead of placing a static condition on the number of peers requesting the same file, we elaborated a complete analysis and experimental evaluation of a dynamic threshold that takes into consideration not only the number of peers, but also their corresponding bandwidths and the size of the shared file. With recourse to previous studies related to the study of HTTP and BitTorrent protocols (Qiu and Srikant, 2004; Wei et al., 2005; Kumar and Ross, 2006; Carbunaru et al., 2014), we proposed accurate estimations of the download times in both protocols and introduced some evaluation metrics to evaluate the efficiency of each of them. These metrics provide accurate estimations of the gain in download time and the amount of data contributed by the peers.

This paper differs from Chaabouni et al. (2014) in that it considers personal clouds with a fixed bandwidth budget constraint. Various related works focused on the allocation of the seed's bandwidth within the BitTorrent swarms when the bandwidth budget is limited (Leon et al., 2014; Peterson and Sirer, 2009; Sharma et al., 2014). However, we believe we are the first to propose an algorithm that manages to distribute the limited cloud bandwidth between the different sets of swarms each downloading the requested content using one of two possible download protocols: HTTP and BitTorrent.

3. Background

In this section, we start by exposing some detailing about the BitTorrent protocol. Later, we propose a definition of personal clouds and detail the general architecture of these systems.

3.1. The BitTorrent protocol

BitTorrent (Cohen, 2003) is a peer-to-peer protocol whose goal is to facilitate fast downloads of files by taking advantage of the upload bandwidth of the peers. A user who wants to share a certain content via BitTorrent has to generate first a *.torrent* file that contains the related meta-data information and a link to a *tracker*. A tracker is a server that assists in the communication between peers interested in the same content. After the generation of the meta-data file, the user has to make the content to be shared available through a BitTorrent node acting as a *seed*. A seed is a node that has a complete copy of a particular content, whereas a *leecher* is one that has only a partial

[1] https://wiki.ubuntu.com/UbuntuOne

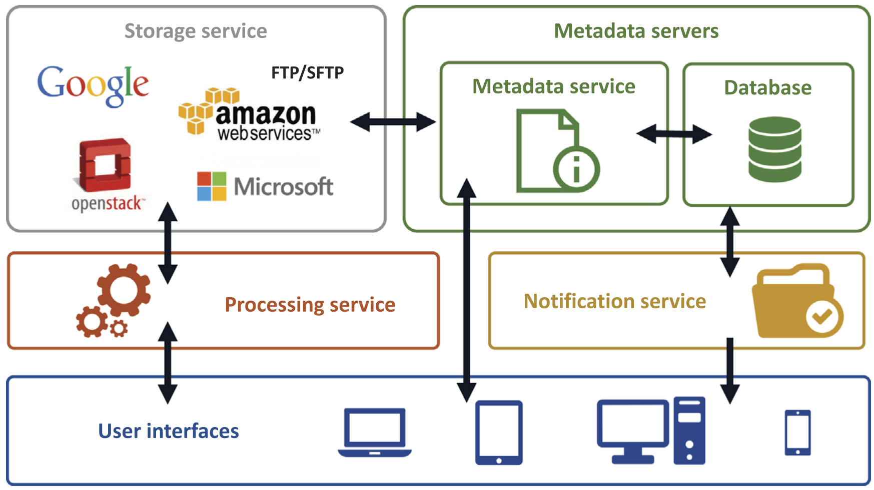


Fig. 1. General architecture of personal clouds.

copy. In this paper, seeds and leechers are simply referred to as *peers* and the set of all the peers sharing the same content is called a *swarm*. Peers interested in a certain content are in charge of getting the corresponding *.torrent* file and then contacting the tracker to acquire a set of other peers sharing the same content. Once these peers are located, it is possible for them to communicate to one another in order to exchange parts of the file among them.[2]

3.2. Personal cloud systems

Definition: A Personal Cloud (PC) is a term generally used to refer to a file hosting service that allows its users to store, synchronize and share content over the Internet. The authors in Garcia-Lopez et al. propose the following definition: "*The Personal Cloud is a unified digital locker for our personal data offering three key services: Storage, Synchronization and Sharing. On the one hand, it must provide redundant and trustworthy cloud data storage for our information flows irrespective of their type. On the other hand, it must provide syncing and file exploring capabilities in different heterogeneous platforms and devices. And finally, it must offer fine-grained information sharing to third-parties* (*users and applications*)".

Architecture: *Figure 1* presents the general architecture of a PC. This figure is inspired from the official architecture of Dropbox (Dropbox, Inc.).

The core elements of a PC are:

- *Meta-data service*: The meta-data servers contain all the meta-data information related to the clients and the files. They can be equipped with a local database where all the meta-data is stored.
- *Storage service*: The storage service or storage back-end refers to the physical locations where the users' data are stored. It can be local, in the form of local storage servers accessed via FTP/SFTP, or external, provided by a third-party (Amazon, Google, etc).
- *Notification service*: The notification service is dedicated to monitoring whether or not any changes have been made to the users' accounts. Whenever a change to any file takes place, the client is notified in order to synchronize these changes.
- *PC clients or user interfaces*: The services offered by personal clouds can be utilized and accessed by physical clients through a number of interfaces, including web interfaces (accessed through web browsers), desktop applications and mobile apps.
- *Processing service*: The processing service is responsible for processing the files and ensuring their delivery to the end-users. To download a file, the client sends an HTTP GET request to the processing service. The latter verifies the existence of the file in the storage nodes and the file is transferred using the HTTP protocol.

4. System architecture

In this paper, we consider a classic personal cloud (Store, Sync and Share functionalities) enriched with extra components that allow inter-client content transfers via BitTorrent. The architecture of the system is presented in Fig. 2. Several components are added to accommodate the BitTorrent behavior, including:

- *Content delivery service*: The content delivery service is also referred to as cloud. Its main role is to process the requests coming from the end-users and ensure the delivery of the files to the corresponding requesters. The main components added, compared to the default architecture (Fig. 1), are:
  - *Coordinator*: The coordinator is the core component of the cloud. It is responsible for managing all the clients' requests and ensuring they are processed correctly. The coordinator is also responsible for the proper management of the cloud's resources.
  - *Seeder nodes*: The seeder nodes are the entities responsible for delivering the requested content from the storage back-end servers to the end-users. To each file being distributed corresponds one seeder node. In our paper, we refer to these seeder nodes as *cloud seeds*

[2] The official protocol specification can be found at: http://www.bittorrent.org/beps/bep_0003.html

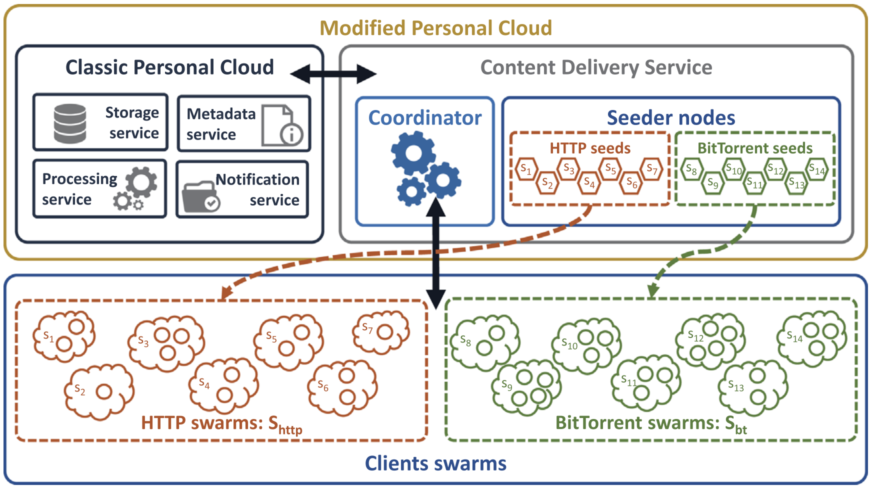


Fig. 2. Global view of the system architecture.

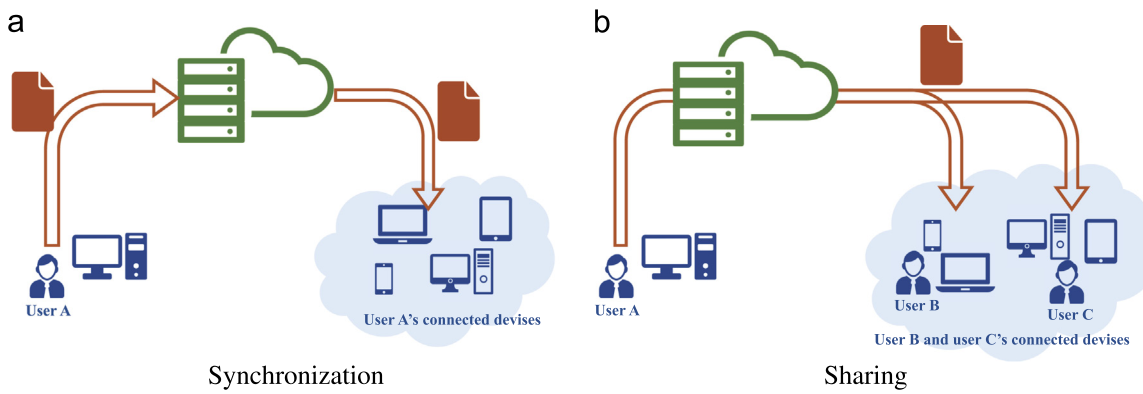


Fig. 3. Synchronization and sharing in personal cloud systems.

or *seeds*. We distinguish two types of seeds: *HTTP seeds* and *BitTorrent seeds* depending on the algorithm adopted to distribute the corresponding requested content to end-users.

- *Clients swarms*: All the end-user peers are organized into swarms. We define a swarm by the set of peers that are requesting the same file. If a file is being downloaded by a single peer, we consider it as a single-peer swarm. This means that, at a given time, there are as many swarms as the number of files being downloaded (to each file corresponds only one swarm and one seeder node). In our model, we distinguish between two types of swarms:
  - *HTTP swarms*: The HTTP swarms are the swarms whose peers are downloading the corresponding file from HTTP seeds via HTTP. Clearly, these peers are not collaborating with each other, but grouping them in swarms is a simple means of control which will help, later on, in making the switching decision.
  - *BitTorrent swarms*: Similar to HTTP swarms, BitTorrent swarms are the swarms whose peers are downloading the corresponding file from BitTorrent seeds via the BitTorrent protocol. Typically, these swarms are composed of two peers or more. Since the peers are supposed to collaborate between each other with the help of the cloud seed, it makes no sense to have a single-peer BitTorrent swarm.

In our system, all the users files are uploaded using HTTP over an encryption layer of SSL/TLS. To download a file, the client sends an HTTP GET request to the coordinator. The latter verifies the existence of the file in the storage nodes and decides the download protocol to be used: HTTP or BitTorrent. The decision is made based on the load on the seed and the swarms' characteristics. In the case of a HTTP download, a HTTP seeder node is associated with the requested file which will be transferred using HTTP (over SSL/TLS). Otherwise, in the case of a BitTorrent transfer, the coordinator creates a torrent meta-data file and launches a corresponding BitTorrent seed. After that, the recently created *.torrent* file will be transmitted to the corresponding clients who, unaware of all these interactions, will then start downloading the file using the BitTorrent protocol (from the cloud seed and/or from the other clients). Evidently, the "old" clients who arrived before the switch to BitTorrent will also benefit from the switch if they did not finish the download. In fact, when an "old" client requests a new part of the file to be downloaded, he will realize that the transfer protocol has changed and will automatically adapt to the new one. Thus, each "old" client will join the swarm with the pieces he already has, which means that he will be probably contributing to the swarm as soon as he switches to BitTorrent in a very transparent way.

This approach can applied widely in any cloud-based system. Personal clouds are the most appropriate for this proposal since the developers can tune the client's implementation to extend them with the BitTorrent functionality. The two following common file distribution scenarios could benefit from our hybrid download strategy:

1. *Synchronization*: User A is adding a new file *f* to his personal account. During the synchronization process, the same file will be download by all the other synchronized devices of the user (Fig. 3(a)).

2. *Sharing*: User A is sharing a file *f* with other users. In this case, the file will be downloaded by all the synchronized devices of the users (Fig. 3(b)).

These scenarios are quite common in personal clouds. In Drago et al. (2012), the authors analyzed a dataset based the behavior of real users of Dropbox. They noticed that about 40% of the households had more than one device linked to the service and that most of these households had up to 4 devices connected. Moreover, about 60% of these households with more than one device have at least one shared folder.

In this paper, we use a trace of the Ubuntu One system which is a personal cloud that was operated by Canonical Ltd. Even though this service was mainly for data backup, we found that more than 11% of the files were downloaded more than once which corresponds to more than 30% of the measured download operations. More details about the trace are provided in Section 6.

4.1. Security concerns

Like most of the personal cloud systems, all the client-server communication in our system is over HTTPS, which protects against eavesdropping and tampering with the contents of the communication. To ensure data confidentiality when the transfer protocol is BitTorrent, we deploy a one-shot symmetric encryption mechanism that uses unique keys to encrypt the files that will be transferred via BitTorrent. In fact, when the decision of using BitTorrent for a given file *f* is made, the cloud creates a corresponding metadata file (*.torrent* file) and assigns to *f* a unique one-shot symmetric encryption key *K* (such as a DES or AES key). This key, along with the *.torrent* file are sent to each of the requesters via HTTPS. The file *f* is encrypted by the server using *K* before being sent to the requesters via BitTorrent. More details are provided in Fig. 4.

When a device completes the download, it simply decrypts the protected file with the received key and the synchronization process terminates. Note that integrity is already guaranteed by BitTorrent itself, so no additional hash computations are needed.

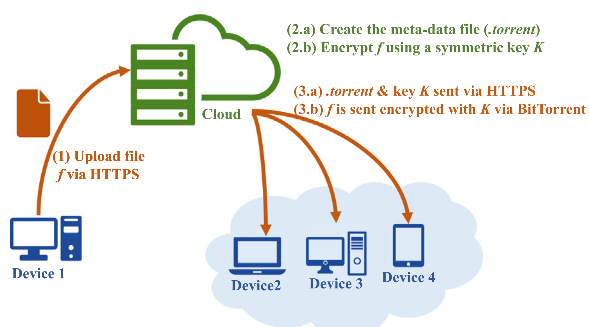


Fig. 4. Encryption mechanism with BitTorrent: Before being sent to Device 2, 3 and 4 via BitTorrent, the file *f* is encrypted on the server's side using a one-shot symmetric key *K*. The *.torrent* metadata file is sent, along with the encryption key *K* via HTTPS.

Table 1
Table of notations.

W	The cloud's upload budget limit
S	The set of all active swarms
S_{http}	A subset of S that corresponds to the set of the swarms downloading the files via HTTP
S_{bt}	A subset of S that corresponds to the set of the swarms switched to BitTorrent
s	A swarm $s = (P_s, f_s, w_s, isBT_s)$ is identified by the set of the peers forming it P_s, the file being shared f_s, the corresponding amount of allocated cloud bandwidth w_s and a boolean variable $isBT_s$ that indicates the download protocol
P_s	Set of all the peers in s. $P_s = \{(u_p, d_p), \forall p \in P_s\}$ where u_p and d_p are respectively the upload and download speeds of a given peer $p \in s$
f_s	File requested by the peers in P_s
w_s	Amount of cloud bandwidth allocated to the swarm s
$isBT_s$	Boolean variable that indicates the download protocol adopted by the peers in s. $isBT_s = True$, if peers in P_s are downloading f_s via BitTorrent and $isBT_s = False$, otherwise
F_s	Size of the requested file f_s
L_s	Number of peers in P_s ($L_s = \vert P_s \vert$)
D_s	Aggregated download speed of all the peers in P_s ($D_s = \sum_{p \in P_s} d_p$)
$d_{min,s}$	The download speed of the slowest peer in P_s ($d_{min,s} = \min_{\forall p \in P_s} d_p$)
u_s	The average upload speed of all the peers in $P_s (u_s = \sum_{p \in P_s} \frac{u_p}{L_s})$
η_s	The effectiveness of file sharing. This variable was first introduced in Qiu and Srikant (2004) and reused later in Chaabouni et al. (2014). η_s takes real values in [0, 1] where 1 means maximum effectiveness while a value of 0 signals the absence of collaboration between peers
α_{bt}	The overhead related to the start-up phase in BitTorrent transfers
τ	The QoS constraint that defines the switching point from HTTP to BitTorrent

5. Bandwidth allocation: problem and solution

5.1. Problem description

In allocating the cloud's bandwidth, our main goal is to minimize the upload bandwidth w_s allocated to a swarm $s \in S$ while taking into consideration the overall quality of service. These allocations should be non-negative ($w_s \geq 0,\ \forall s \in S$) and their sum should not exceed the cloud's upload budget limit ($\sum_{s \in S} w_s \leq W$). The bandwidth allocation strategy follows these two rules:

Rule 1. *HTTP is the default download protocol and each swarm is allocated a share of the cloud's bandwidth equal to its demand.*

Rule 2. *The cloud can decide to switch the download protocol for a given swarm s from HTTP to BitTorrent if it (the cloud) will save in bandwidth and if this change of protocol will not jeopardize the quality of service constraint related to the download time of the peers in s.*

Rule 1 defines HTTP as the default download protocol and sets the share of the cloud's bandwidth allocated to s to be equal to the aggregated download speeds of the peers in s ($w_s = D_s$). When it is possible to gain in bandwidth, the cloud can decide to switch the download protocol from HTTP to BitTorrent, as stated in Rule 2. This change of protocols is fixed by a quality of service constraint τ. This constraint defines the degree of improvement (or degrade) in download time that is accepted when considering the switch to BitTorrent. For instance, a $\tau = 0.2$ requires an improvement of a least 20% in download time that a swarm would gain if it adopts BitTorrent rather than HTTP. A negative value of τ, such as -0.5 means that losses in download time up to 50% are accepted. The bandwidth allocated to BitTorrent swarms should be the minimal that satisfies the switching constraint. As the swarms evolve over time with new peers joining and leaving, we will need to adapt the assignments and allocations accordingly since the cloud's upload speed is limited by W. The complete notation is presented in Table 1.

To better understand the problem, it is important to introduce some parameters that will interfere in making the decision about the bandwidth allocation. These parameters serve to compare the performance of using HTTP or BitTorrent to distribute a given file shared between a set of peers.

We consider the case of a swarm s composed of L_s distinct peers requesting the same file f_s from the same source, called the cloud seed (or simply the seed). We denote by w_s the allocated share of the cloud's upload bandwidth reserved to s, F_s the size of the shared file f_s, u_s the average upload speed of the peers in the s and by $d_{min,s}$ the download speed of the slowest peer among them (see Fig. 5 for more details).

In the following subsections, we will present some formulas related to the estimation of the download times in HTTP and BitTorrent (respectively T_{http} and T_{bt}). We will also define the gain percentage *Gain* and estimate the minimum cloud upload bandwidth needed to satisfy the QoS constraint for s by solving the equation $Gain(w_s^{bt}, s) \geq \tau$.

5.1.1. Download time in HTTP T_{http}

In the case f_s is distributed via HTTP, the distribution time T_{http} is limited by the download speed of the slowest peer $d_{min,s}$ or the seed bandwidth w_s divided equally between the L_s clients. It can be defined as follows:

$$T_{http}(w_s, s) = \frac{F_s}{\min\left\{d_{min,s}, \dfrac{w_s}{L_s}\right\}}. \tag{1}$$

5.1.2. Download time in BitTorrent T_{bt}

When f_s is distributed via BitTorrent, then the distribution time T_{bt} depends on the download speed of the slowest peer $d_{min,s}$, the aggregated upload bandwidth of all the nodes divided equally between all the L_s leechers, and the cloud's allocated share w_s. T_{bt} has been studied before in Kumar and Ross (2006) and an approximation of the distribution time was given. In Chaabouni et al. (2014), we extend that approximation and propose an accurate estimation of T_{bt} that takes into consideration the overheads related to the nature of the protocol, as follows:

$$T_{bt}(w_s, s) = \frac{F_s}{\min\left\{d_{min,s}, \dfrac{w_s + \eta_s . u_s . L_s}{L_s}, w_s\right\}} + \alpha_{bt}, \tag{2}$$

where α_{bt} is the overhead related to the start-up phase, and η_s measures the effectiveness of file sharing for s. For more details about α_{bt} and η_s, refer to Chaabouni et al. (2014). Eq. (2) was validated in Chaabouni et al. (2014) using different bandwidth settings and different file sizes, and was proven to be accurate even with small files.

5.1.3. The gain ratio

Sometimes the use of BitTorrent may incur a longer download time compared to HTTP especially for small files. The main challenge is to decide when it is worth switching to BitTorrent. The key element in making the decision is the gain in download time which represents the difference in download time between HTTP and BitTorrent. To this extend, we introduced in Chaabouni et al. (2014) the gain ratio

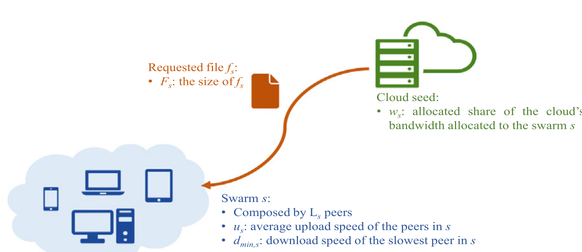


Fig. 5. File distribution scenario.

which measures the improvement in terms of download time between client–server and peer-assisted systems, as follows:

$$Gain(w_s, s) = \frac{T_{http}(w_s, s) - T_{bt}(w_s, s)}{T_{http}(w_s, s)}.$$

The gain can take different values which can be either negative, positive or equal to zero. A positive (respectively negative) gain ratio equal to x (respectively $-x$) means that downloading the file via BitTorrent entails a gain (respectively a loss) of $100\,x\%$ in download time compared to HTTP. A gain ratio equal to zero indicates that both protocols (BitTorrent and HTTP) have the same estimated download time.

We derived in Chaabouni et al. (2014) the analytic equation of the gain based on the values of the divisors of T_{http} and T_{bt}, which are respectively $\min\left\{d_{min,s}, \frac{w_s}{L_s}\right\}$ and $\min\left\{d_{min,s}, \frac{w_s+\eta_s\,L_s\,u_s}{L_s}, w_s\right\}$, as follows:

$$Gain(w_s, s) = \begin{cases} -\dfrac{\alpha_{bt}\,d_{min,s}}{F_s}, & \text{if } d_{min,s} \le \dfrac{w_s}{L_s} \text{ and } d_{min,s} \le \min\left\{\dfrac{w_s+\eta_s\,u_s\,L_s}{L_s}, w_s\right\} \\ 1-\dfrac{w_s}{L_s\,d_{min,s}} - \dfrac{\alpha_{bt}\,w_s}{F_s\,L_s}, & \text{if } \dfrac{w_s}{L_s} \le d_{min,s} \text{ and } d_{min,s} \le \min\left\{\dfrac{w_s+\eta_s\,u_s\,L_s}{L_s}, w_s\right\} \\ 1-\dfrac{w_s}{w_s+\eta_s\,u_s\,L_s} - \dfrac{\alpha_{bt}\,w_s}{F_s\,L_s}, & \text{if } \dfrac{w_s+\eta_s\,u_s\,L_s}{L_s} \le \min\{d_{min,s}, w_s\} \\ 1-\dfrac{1}{L_s} - \dfrac{\alpha_{bt}\,w_s}{F_s\,L_s}, & \text{if } w_s \le \min\left\{d_{min,s}, \dfrac{w_s+\eta_s\,u_s\,L_s}{L_s}\right\}. \end{cases} \quad (3)$$

5.1.4. Solving the equation $Gain(w_s^{bt}, s) \overset{??}{\ge} \tau$

In order to calculate the minimum amount of cloud bandwidth needed to ensure that the switching condition $Gain(w_s^{bt}, s) \ge \tau$ is satisfied, it is essential to reverse the gain formulation (Eq. (3)). To this extend, we study the behavior of the gain formulas when ${w_s}^{bt}$ varies. Based on this constraint, we identify two exhaustive cases in which the gain equations are monotonically decreasing. For each case, we deduce the reversed equations of the gain, interval per interval, as follows[3]:

- *Case A*: $(L_s-1)d_{min,s} \ge L_s\,\eta_s\,u_s$: the average download speed of the peers in the swarm s is higher than the upload bandwidth that the whole swarm can provide:

$$w_s^{bt} = \begin{cases} L_s\,d_{min,s}, & \forall \tau \in \left[-\infty, -\dfrac{\alpha_{bt} d_{min,s}}{F_s}\right] \\ \dfrac{(1-\tau)F_s\,L_s\,d_{min,s}}{F_s + d_{min,s}\,\alpha_{bt}}, & \forall \tau \in \left[-\dfrac{\alpha_{bt}\,d_{min,s}}{F_s}, \dfrac{\eta_s\,u_s}{d_{min,s}} - \dfrac{\alpha_{bt}(d_{min,s}-\eta_s\,u_s)}{F_s}\right] \\ \dfrac{\sqrt{a^2b^2-2abc+4ab+c^2}-ab-c}{2\,b}, & \forall \tau \in \left[\dfrac{\eta_s\,u_s}{d_{min,s}} - \dfrac{\alpha_{bt}(d_{min,s}-\eta_s\,u_s)}{F_s}, 1-\dfrac{1}{L_s} - \dfrac{\alpha_{bt}\,\eta_s\,u_s}{(L_s-1)F_s}\right] \\ \dfrac{F_s[L_s(1-\tau)-1]}{\alpha_{bt}}, & \forall \tau \in \left[1-\dfrac{1}{L_s} - \dfrac{\alpha_{bt}\,\eta_s\,u_s}{(L_s-1)F_s}, 1-\dfrac{1}{L_s}\right] \\ \nexists, & \forall \tau \in \left[1-\dfrac{1}{L_s}, +\infty\right] \end{cases}$$

  where:

$$a = \eta_s\,L_s\,u_s, \quad b = \frac{\alpha_{bt}}{F_s\,L_s} \quad \text{and } c = \tau \quad (4)$$

- *Case B*: $(L_s-1)d_{min,s} \le L_s\,\eta_s\,u_s$: the average download speed of the peers in the swarm s is lower than the upload bandwidth the whole swarm can provide:

$$w_s^{bt} = \begin{cases} L_s\,d_{min,s}, & \forall \tau \in \left[-\infty, -\dfrac{\alpha_{bt}\,d_{min,s}}{F_s}\right] \\ \dfrac{(1-\tau)F_s\,L_s.d_{min,s}}{F_s + d_{min,s}\,\alpha_{bt}}, & \forall \tau \in \left[-\dfrac{\alpha_{bt}\,d_{min,s}}{F_s}, 1-\dfrac{1}{L_s} - \dfrac{\alpha_{bt}\,d_{min,s}}{F_s\,L_s}\right] \\ \dfrac{F_s[L_s(1-\tau)-1]}{\alpha_{bt}}, & \forall \tau \in \left[1-\dfrac{1}{L_s} - \dfrac{\alpha_{bt}\,d_{min,s}}{F_s\,L_s}, 1-\dfrac{1}{L_s}\right] \\ \nexists, & \forall \tau \in \left[1-\dfrac{1}{L_s}, +\infty\right] \end{cases}$$

5.2. Bandwidth allocation and protocol management algorithm

In this section we present our bandwidth distribution and protocol management algorithm. This algorithm aims to minimize the cloud's allocated bandwidth among the seeder nodes, while respecting the QoS constraint. We remind that a seeder node is an entity responsible for distributing a given file to the corresponding set of clients. The share of the cloud's upload bandwidth allocated to each seeder node should verify the constraints of the problem previously explained in Section 5.

[3] For the complete proof of the solution, refer to Appendix A.

In addition to the bandwidth allocations, the algorithm is also responsible for evaluating for each swarm the most suitable content distribution protocol: HTTP or BitTorrent. A swarm would switch to BitTorrent if the following conditions are satisfied:

1. The number of clients in the swarms is higher or equal to 2. In fact, it makes no sense to use BitTorrent with only one client interested in the file.
2. The switch to BitTorrent should satisfy the quality of service constraint τ. This means that BitTorrent can be used only when the gain percentage (Eq. (3)) is higher or equal than τ.
3. The amount of cloud bandwidth allocated in BitTorrent should be smaller than the one using HTTP. This means that the switch will only take place if the cloud would gain in terms of bandwidth.

We proposed in Chaabouni et al. (2014) a simple decision approach that consists in calculating for each swarm $s \in S$, with more than one peer, how much the swarm would gain in terms of download time if BitTorrent is used instead of HTTP. If that gain satisfies the quality of service constraint τ, then BitTorrent is chosen. Otherwise, HTTP is kept as the download protocol. Even though this approach is simple and direct, it has to be further improved in order to satisfy the third switching condition. In this paper, we go a step further and calculate the minimum amount of bandwidth ${w_s}^{bt}$ needed to satisfy the constraint $Gain(w_s^{bt}, s) \geq \tau$ (Section 5.1.4). Thus, instead of comparing $Gain(w_s^{bt}, s)$ and τ, we compare D_s and ${w_s}^{bt}$ and the protocol that requires less bandwidth is chosen.

5.3. Algorithm description

The main goal of our bandwidth distribution and switching algorithm is to optimally manage the cloud's limited bandwidth among the seeder nodes. It is also responsible for evaluating for each requested file the most suitable content distribution model: client–server or peer-assisted, based on the current demand load. Each active seeder node in the system is associated with a swarm of clients that are interested in the same file. It is important to remind here that the default bandwidth distribution protocol is HTTP, and BitTorrent can be also used when the switching conditions previously stated are satisfied. The swarms whose peers are using HTTP as a transfer protocol are referred to as *HTTP swarms* (S_{http} is the set of HTTP swarms) and the ones with peers downloading via BitTorrent are labeled as *BitTorrent swarms* (S_{bt} is the set of BitTorrent swarms).

The algorithm is executed whenever a change affects a swarm $s^* \in S$. This change can be related to a modification in one or more of the parameters of a certain swarm. This change can be due to one or more of the following cases:

- A new peer p^* wants to download a file f_{s^*}. If the file is already requested by other peers, then p^* will be added to the existing swarm s^*. Otherwise, a new swarm s^* will be created containing a single peer p^*.
- A peer p^* leaves a swarm s^*. If p^* was not the only peer in the swarm, then the modified swarm will contain a list of the other remaining peers. If p^* was the last peer in s^*, then s^* will be removed from S.
- The upload or download speed of one or more of the peers in s^* changes.

The algorithm requires the following input parameters: the set of all current swarms S, the swarm affected by the change s^*, the cloud's upload bandwidth budget limit W and the switching constraint τ.

Algorithm 1. Bandwidth distribution and switching algorithm.

Input S ▹the set of all the current swarms ($S = S_{http} \cup S_{bt}$)
Input s^* ▹swarm affected by a change
Input W ▹the cloud's upload bandwidth budget limit
Input τ ▹the switching constraint
if $L_{s^*} = 1$ **then** ▹s^* is a single-peer swarm
  $w_{s^*} = D_{s^*}$
else if $L_{s^*} > 1$ **then** ▹s^* has more than one peer
  if $s^* \in S_{http}$ **then** ▹s^* is a HTTP swarm
    calculate $w_{s^*}^{bt}$ using Eq. (4)
    if $w_{s^*}^{bt} \leq D_{s^*}$ **then** ▹switching to BitTorrent
      switch the transfer protocol from HTTP to BitTorrent
      $isBT_{s^*} = True$ ▹mark s^* as a BitTorrent swarm
      $w_{s^*} = w_{s^*}^{bt}$
    else ▹not switching to BitTorrent
      $w_{s^*} = D_{s^*}$
    end if
  else ▹$s^* \in S_{BitTorrent}$, s^* is a BitTorrent swarm
    $w_{s^*} = w_{s^*}^{bt}$ calculated using Eq. (4)
  end if
else ▹$L_{s^*} = 0$, s^* no longer exists
  remove s^* from S
  if $\sum_{s \in S} w_s + w_{s^*} = W$ **then** ▹the cloud was overloaded
    for each s **in** S_{bt} **do**
      $w_s = w_s + \frac{w_s}{\sum_{s \in S_{bt}} w_s} w_{s^*}$ ▹redistribute w_{s^*} to the BitTorrent swarms
    end for
  end if

23: **end if**
24: **if** $\sum_{s \in S} w_s > W$ **then**
25: **for each** s **in** S **do**
26: $w_s = \frac{w_s}{\sum_{s \in S} w_s} W$ ▹scale down all the bandwidth shares
27: **end for**
28: **end if**

Using these input parameters, the algorithm identifies for each swarm the most suitable download protocol (HTTP or BitTorrent) and calculates the amount of bandwidth to be allocated to the corresponding seed, as follows:

- If s^* is a single-peer swarm ($L_{s^*} = 1$), then the cloud allocates to s^* a share of bandwidth equal to its download capacity: $w_{s^*} = D_{s^*}$ (*lines* 1 and 2). In this case, the file will be distributed directly from the cloud seed to the single-peer using HTTP.
- If the number of peers in s^* is strictly higher than 1 (*lines* 3–12), then there are two possible cases:
  - If the peers in s^* are using HTTP to download f_{s^*} ($isBT_{s^*} = False$), the algorithm verifies if it is worth it to switch to BitTorrent. To do so, $w_{s^*}^{bt}$ is calculated according to Eq. (4). We remind that $w_{s^*}^{bt}$ measures the amount of seed bandwidth required to verify the quality of service constraint τ when using BitTorrent for s^*. The algorithm compares later this bandwidth (w_{s^*}) with the bandwidth allocated by default to the swarm (which is equal to D_{s^*}).
    - If the bandwidth required using BitTorrent is smaller than the one allocated by default ($w_{s^*}^{bt} \leq D_{s^*}$), then the download protocol more suitable for s^* is BitTorrent (*lines* 4–9). In this case, a.torrent file associated to f_{s^*} is created and a BitTorrent seed is launched in the cloud. All the peers in s^* have to download the .torrent file recently created and then can start downloading f_{s^*} via BitTorrent.
    - If the use of BitTorrent requires more bandwidth than HTTP, then it is not worth switching to BitTorrent. In this case, the cloud allocates a share of bandwidth equal to D_{s^*} (*line* 11).
  - If s^* has already switched to BitTorrent, then the algorithm recalculates $w_{s^*}^{bt}$: the bandwidth needed to maintain the quality of service constraint τ. The amount of bandwidth allocated to s^* is equal to $w_{s^*}^{bt}$.
- If s^* is an empty swarm ($L_{s^*} = 0$), then the swarm is removed from the swarms' list. If the cloud was overloaded before the removal of s^*, then the amount of bandwidth that was previously allocated to s^* is redistributed among the BitTorrent swarms (*lines* 18–22). This will prevent the cloud's bandwidth from being underutilized and will boost the distribution of the files among the BitTorrent swarms.

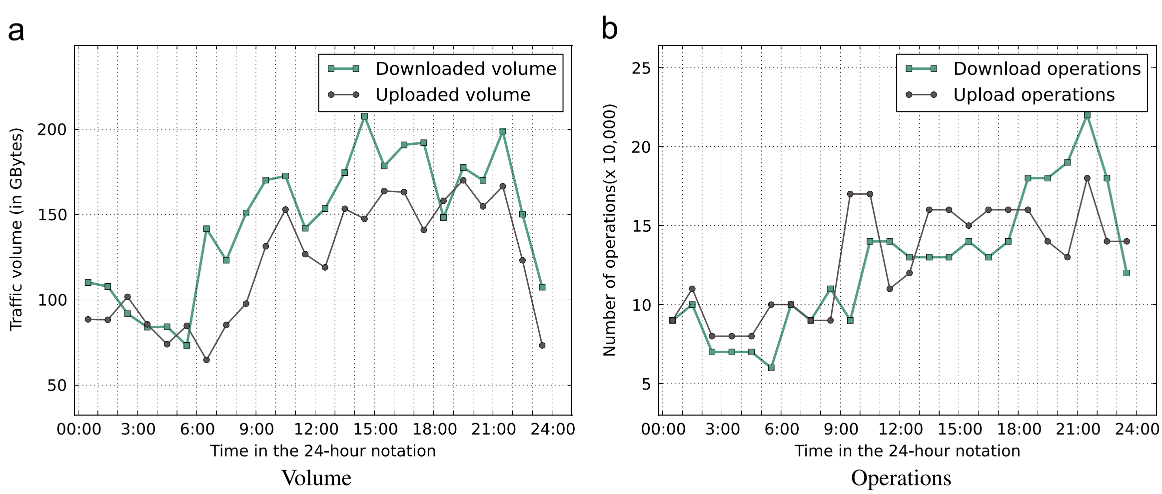


Fig. 6. Total uploaded and downloaded volume and the number of upload and download operations per hour during the day January 21st, 2014 for the UB1 system.

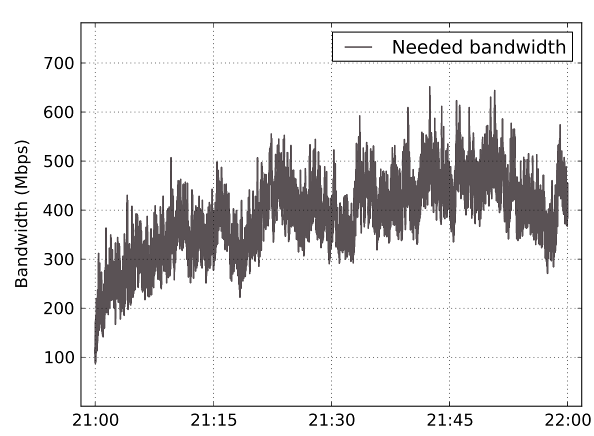


Fig. 7. Needed upload bandwidth over time during the peak hour.

Table 2
Percentages of operations with gains and losses in download time resulted by the algorithm compared to pure HTTP use. The cloud upload bandwidth budget limit is $W = 300$ Mbps.

Results	$\tau_1 = -0.2$ (%)	$\tau_2 = 0$ (%)	$\tau_3 = 0.2$ (%)	$\tau_4 = 0.4$ (%)
% of operations with gain	82.89	82.9	83.43	83.53
% of operations with loss	2.23	2.31	2.48	2.85
% of operations with no difference	14.88	14.79	14.09	13.62
Total %	100	100	100	100

Table 3
Total sum of all the download times for all the operations and the net gain percentage for the algorithm applied on the one-hour sample of the UB1 trace. The cloud upload bandwidth budget limit is $W = 300$ Mbps.

Results	$\tau_1 = -0.2$	$\tau_2 = 0$	$\tau_3 = 0.2$	$\tau_4 = 0.4$
sum_http_hours (h)	2450.2	2450.2	2450.2	2450.2
sum_algo_hours (h)	2000.98	1997.05	1952.73	1906.56
net_gain_hours (h)	449.22	453.15	497.47	543.64
net_gain_%	**18.33%**	**18.49%**	**20.3%**	**22.19%**

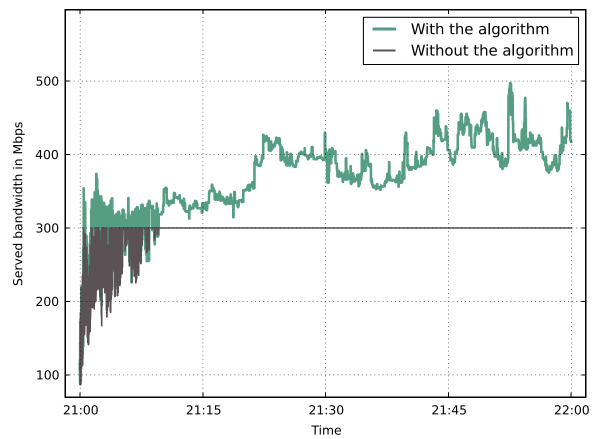


Fig. 8. Amount of bandwidth served to the clients with and without the algorithm. The extra served bandwidth with the algorithm comes from the peers involved in BitTorrent swarms. Settings: $W = 300$ Mbps and $\tau = 0.4$.

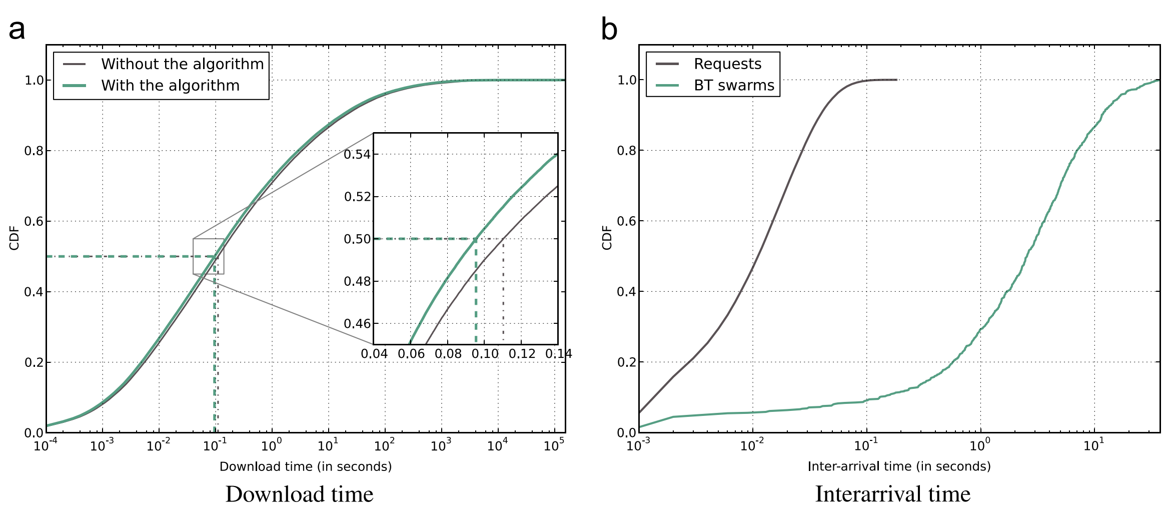


Fig. 9. CDF download times and inter-arrival time of requests and BitTorrent swarms with and without the algorithm. Settings: $W = 300$ Mbps and $\tau = 0.4$.

When the number of simultaneous requests becomes high, the seed might be unable to serve all the swarms at their full speed. In such a case, the cloud has to scale down all the bandwidth allocations proportionally to the demand (*lines* 24–28).

Complexity of the algorithm: Our bandwidth distribution and switching algorithm has a complexity of $O(n)$ which is linear with the number of current swarms. The coordinator only keeps in memory the state of the swarms during the iterations. This corresponds to $n \times k$ units of storage where n is the maximum number of simultaneous swarms and k is the size, in unit of storage, required to store the essential information about a current swarm. k is rather small (compared to the size of the files) and it depends on the number of peers in the swarm and the storage space required to store the information of each one of these peers.

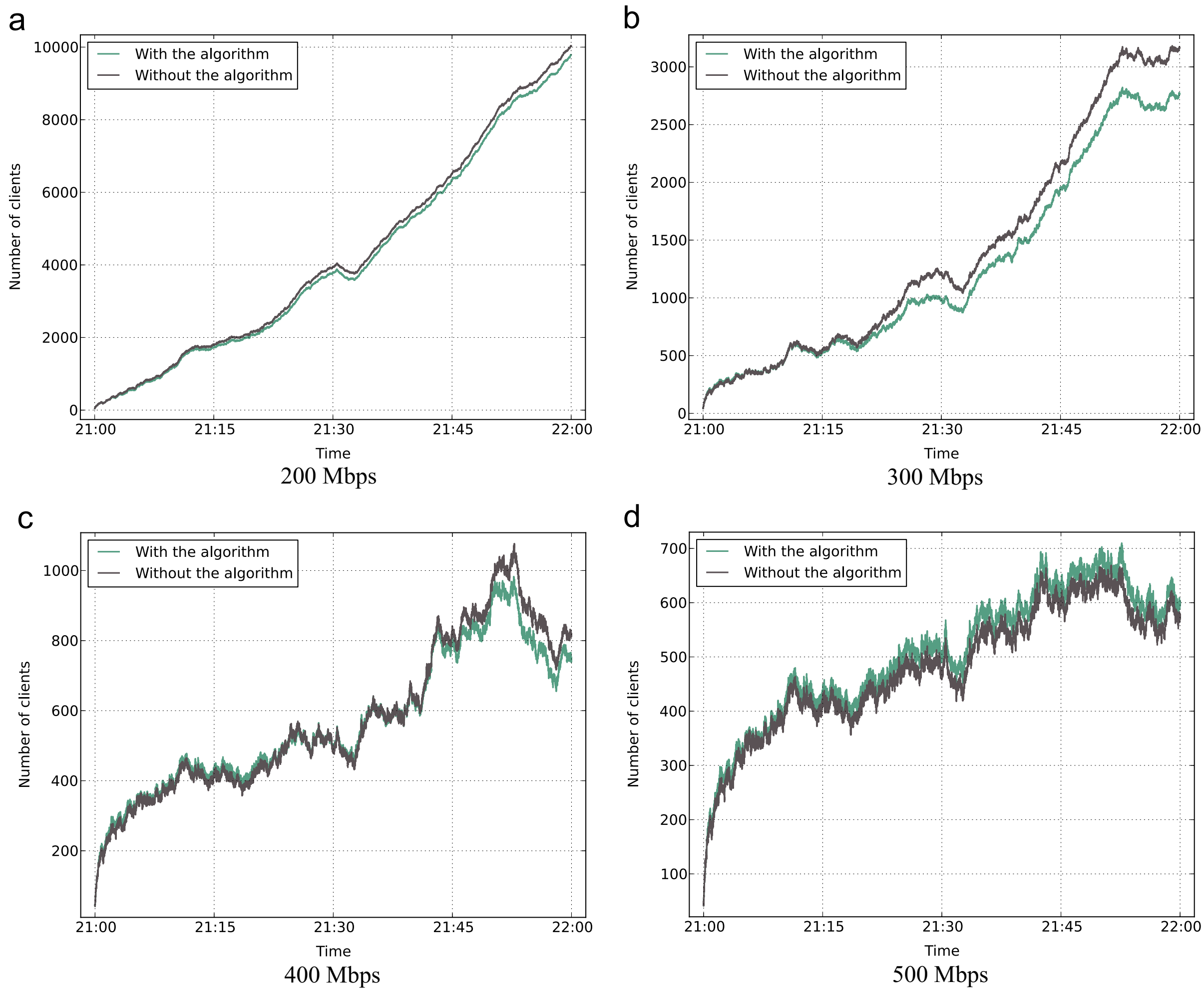


Fig. 10. Number of simultaneous clients present in the system with and without the algorithm. Settings: w=200, 300, 400, and 500 Mbps; τ=0.4.

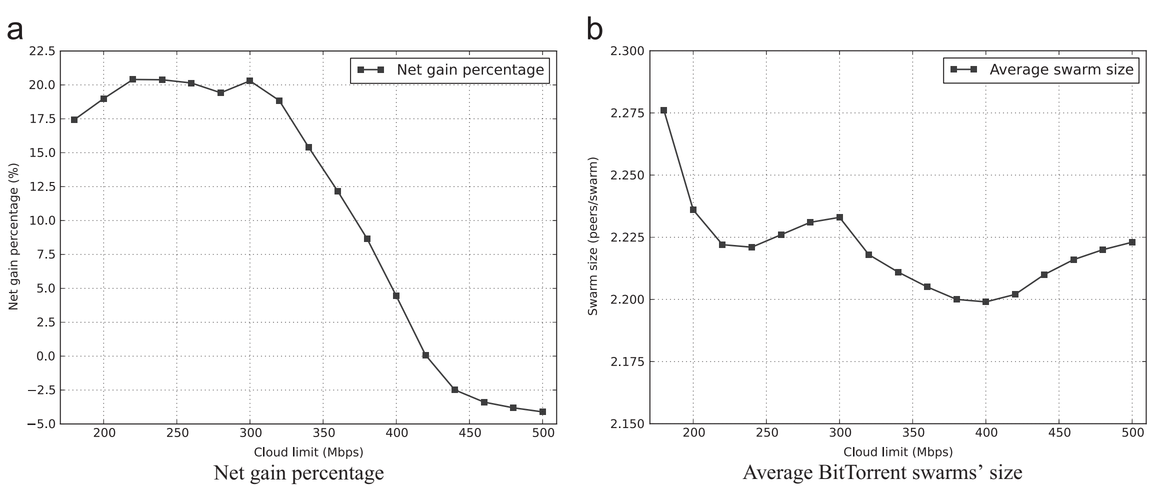


Fig. 11. Net gain percentage and average size of the BitTorrent swarms for different cloud bandwidth limits ranging between 180 and 500 Mbps. The switching constraint considered here is τ=0.2.

6. Validation of the algorithm: application to the personal cloud scenario

In order to evaluate the performance of the proposed algorithm, we take advantage of a real trace of the Ubuntu One personal cloud system (Tinedo et al., 2015). In this context, we implement two simulators: the first simulates the default behavior of the cloud where all the download requests are treated individually and the files are distributed via HTTP. The second simulates the bandwidth distribution and protocol management algorithm. We compare later the results of using each of the approaches on the trace.

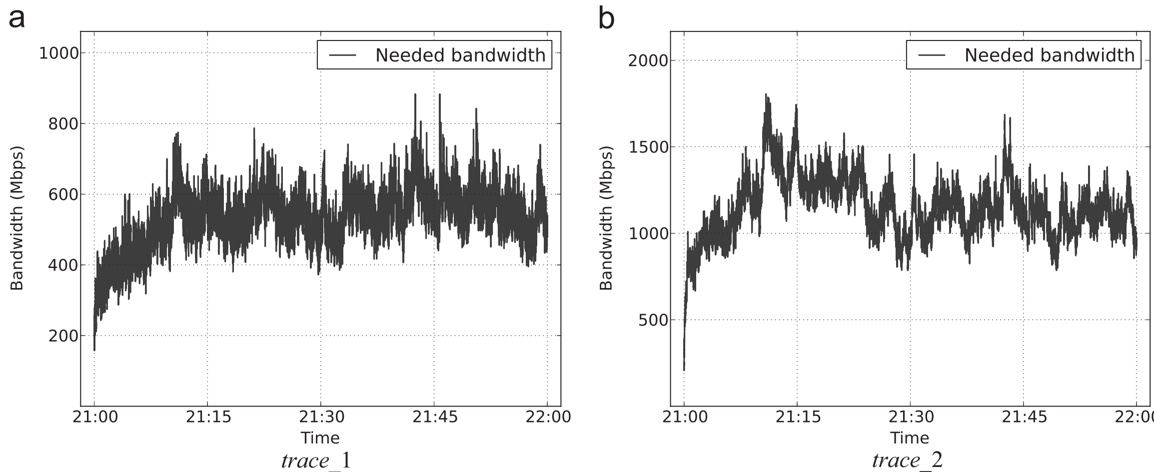


Fig. 12. New bandwidth requirements of the modified traces.

6.1. The Ubuntu One trace

In our validation, we use a real trace of the Ubuntu One (UB1) system. The trace was provided by *Canonical Ltd.*[4] in the context of the CloudSpaces project.[5] The logs were collected for about a month from their servers located in London, based on the behavior of real users. Each line of the trace represents an upload or download operation performed by one user on a given file. For the sake of privacy, files and user real identifiers are presented in the form of unique hash codes. For each operation, several information were collected, including: the timestamp, the type of operation ("up" or "down"), the hash and size of the file in question, the user's hash identifier and the corresponding upload and download bandwidths. We filtered the original trace and focused on the upload and download operations that were performed during a random day of the trace (January 21st, 2014). For that day, 6,331,131 operations performed by 33,257 distinct client on 4,095,057 unique files were logged.

Figure 6 shows the total uploaded and downloaded volume along with the total number of upload and download operations measured per hour during the whole day. The hours are logged according to the Greenwich Mean Time (GMT). We notice that the peak with the highest number of upload and download operations corresponds to the hour between 21:00 and 22:00 GMT.

To facilitate the validation process, we focus only on that peak hour, but we believe the results would be similar for the whole trace. Since we aim to manage efficiently the upload speed of the cloud, we filter the one-hour sample to keep only 225,514 download operations related to 173,756 distinct files. The average file size in the sample is about 1 MB, 988.26 KB to be precise. We notice in that sample that about 68.62% of the operations correspond to single downloads. Single downloads are operations related to files downloaded only once. These files account for about 89% of the total files downloaded between 21:00 and 22:00. This means that only 31.38% of the operations correspond to multiple downloads of the same file and that only 11% of the files were downloaded more than once. This signifies that only 31.38% of the operations are potential candidates for switching to BitTorrent.

Focusing more on that peak hour, we calculate how much bandwidth should be provided by the cloud seed in order to satisfy the need of all the requesting peers if the download protocol was HTTP. To do so, we went through the trace tracking the active swarms at each timestamp and summing up all the download capacities of the active peers. With each new download request, we updated the amount of data left to be downloaded by each peer. Once a peer has finished downloading the file, it was removed from the active peers list. The download times were calculated according to Eq. (1). We plot the resulting amount of needed bandwidth in Fig. 7.

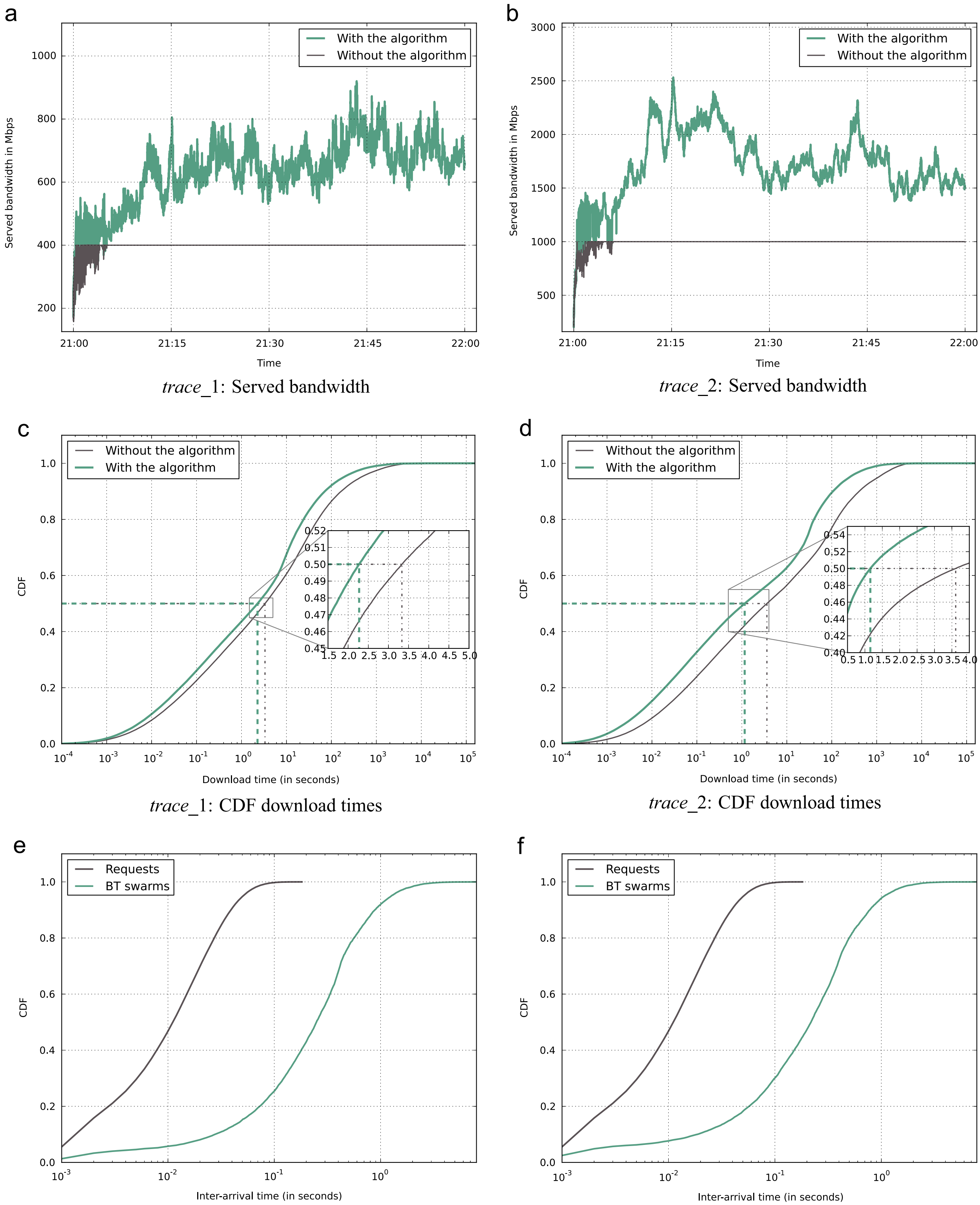
This figure will be useful later to set a potential limit on the cloud seed's bandwidth when evaluating the algorithm's efficiency. It shows that the total need in cloud bandwidth does not exceed 650 Mbps. So, when evaluating the algorithm, it would be better to vary the cloud limit $W \in]0, 650[$ in order to measure the effect of the seed's capacity on the algorithm's performance.

6.2. Experimental settings

To evaluate the efficiency of our proposal, we developed a Python[6] script that simulates the bandwidth distribution and switching algorithm (Algorithm 1) and logs the results in two different log files. The first log file is related to the seed: it keeps a log of the current state of the seed. At each timestamp, several parameters are logged including: the amount of needed bandwidth, the amount of bandwidth served by the seed and the current number of swarms and clients. The second log keeps track of the start time and end time of each download. The download times are updated at each timestamp according to Eqs. (1) and (2), for HTTP and BitTorrent swarms, respectively.

In order to evaluate the results, we also developed another script that simulates the default behavior of the seed in which each download operation is treated individually and the download times are calculated according to Eq. (1). This simulator also keeps similar logs as the algorithm simulator in order to facilitate the comparison of the approaches.

[4] Canonical Ltd: http://www.canonical.com
[5] FP7 CloudSpaces Project: http://www.cloudspaces.eu
[6] Python Software Foundation: http://www.python.org



*trace*_1: Served bandwidth

*trace*_2: Served bandwidth

*trace*_1: CDF download times

*trace*_2: CDF download times

*trace*_1: CDF inter-arrival times

*trace*_2: CDF inter-arrival times

Fig. 13. Results using the modified traces.

Table 4
Comparison of the results with the three different traces. The experiment settings are the following: τ=0.4 for all the traces, W is 300 Mbps for the original trace, 400 Mbps for *trace_1* and 1000 Mbps for *trace_2*.

Results	Original trace	*trace_1*	*trace_2*
Number of operations affected by the change	0	69,286	65,992
(% of the total number of operations)	(0%)	(30.72%)	(29.26%)
Number of operations switched to BitTorrent	1734	36,727	43,997
(% of the total number of operations)	(0.76%)	(16.28%)	(19.5%)
Number of BitTorrent swarms created	786	9,324	10,763
Average BitTorrent swarm size (in peer/swarm)	2.206	3.93	4.08
Average inter-arrival time of BitTorrent swarms (s)	4.570	0.386	0.334
Average download time without the algorithm (s)	39.11	106.09	186.82
Average download time with the algorithm (s)	30.43	52.77	61.08
net_gain_%	**22.19%**	**50.26%**	**67.30%**

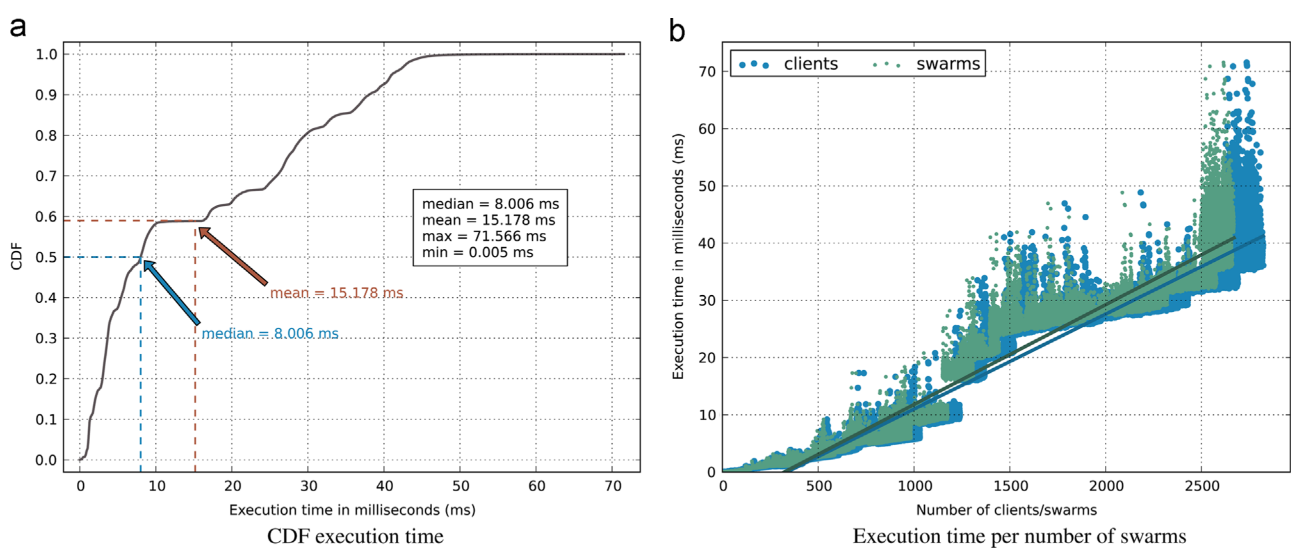


Fig. 14. Algorithm's performance during the trace simulation.

6.3. Results

We exploit the previously described trace sample of UB1 and re-simulate the arrival pattern of the peers to validate our approach. We run both simulators with a wide combination of τ and W values and collect the logs of each experiment. Then, we evaluate our algorithm comparing the results with the ones obtained using the default strategy with the same bandwidth limits.

First of all, we run the simulator fixing the upload bandwidth budget at 300 Mbps and varying the switching constraint τ. The goal is to get a first idea of the performance of the algorithm. We measure for each simulation, the download time taken by each operation and compare them to the times measured using the HTTP-only simulator with the same budget limit. It is important to note here that the download times are measured in seconds with a precision of one millisecond. We classify the operations into three different categories: operations that have gained in download time with the algorithm, operations that experienced losses and operations whose download time is left unchanged for both approaches.

Table 2 presents the percentages of the operations in each category. We notice that for the three different values of τ, more than 80% of the operations benefited from a gain in download time, about 15% kept the same time and only less 3% of them lost in download time. Even though these percentages are quite good, we need to make sure that the cumulative gains are higher than the losses. To do so, we sum all the download times of all operations for both approaches and calculate the total net gain percentage (*net_gain_%*). *net_gain_%* represents the percentage ratio between the total time gained (or lost) by using the algorithm (*net_gain_hours* = *sum_http_hours* − *sum_algo_hours*) and the total download times using HTTP only (*sum_http_hours*):

$$net\_gain\_\% = \frac{net\_gain\_hours}{sum\_http\_hours} \times 100 = \frac{sum\_http\_hours - sum\_algo\_hours}{sum\_http\_hours} \times 100$$

Table 3 presents the total sum of all the download times of all the download operations and the net gain percentage based on the UB1 one-hour sample. The first row represents the sum of download times using HTTP. It is important to mention here that, for HTTP, this sum depends only on the cloud upload bandwidth budget W. Hence, for the fixed bandwidth W=300 Mbps, it is always equal to 2450.2 h, regardless of the τ constraint. However, the sum of the download times using the algorithm with a given cloud bandwidth limit depends highly on the switching constraint τ. In Table 3, we compare the results with three different τ values.

The first constraint is $\tau_1 = -0.2$: this constraint can be translated as follows: at a certain timestamp, a swarm can switch from HTTP to BitTorrent only if it will only lose less than 20% in download time. Under this constraint, we notice that the algorithm performs better than HTTP with a net gain in the client's download time equal to 18.33%. Next, we make the constraint a little bit stricter and we accept only switches to BitTorrent when the peers in question will only gain in download ($\tau_2 = 0$, no loss is permitted). We notice that the net gain percentage improves slightly. This is because the constraint will prevent swarms with negative gains from switching which will result in an increase of the total amount of net gain hours. Similarly, with the third and fourth constraints $\tau_3 = +0.2$ and $\tau_4 = +0.4$ (switch only if the corresponding peers will gain 20%, respectively 40%, or more gain in download time), the net gain percentage gets higher and reaches more than 20% of the total download time of all peers.

To measure the efficiency of the algorithm for a specific configuration, we fix the switching constraint τ=0.4 and we suppose that the cloud's upload budget limit W=300 Mbps. Figure 8 shows the efficiency of taking advantage of the upload speed of the peers. It compares the amount of bandwidth served by the seed to the clients without using the algorithm (all files are distributed via HTTP) versus the total amount that becomes available when using the algorithm. This latter includes, in addition to the cloud's upload limit, the upload speed of the clients who switched to BitTorrent. We notice that the peers' contribution can reach up to 60% of the total cloud's budget.

The total operations switched to BitTorrent represents only 2.45% of the operations with multiple downloads, which corresponds to less than 1% of the total number of operations, and the average size of the swarms switched to BitTorrent is 2.206 peers per swarm. Despite this limited number of switched operations, we notice in Fig. 9(b) that the download times are reduced using the algorithm. As a matter of fact, the average download time without using the algorithm is 39.11 s. This time is reduced by 22.19% using the algorithm to only 30.43 s. Figure 9(b) presents the CDF of the inter-arrival times of requests and BitTorrent swarms. The average inter-arrival rate of the download requests (time between each arrival of a download request into the system and the next) is 0.0159 s. The average inter-arrival rate of the BitTorrent swarms (time between each creation of a BitTorrent swarm and the next) is 4.5702 s.

After evaluating the general performance of the algorithm, we study the effect of the bandwidth limit W. Figure 10 compares the number of simultaneous clients in the HTTP-only mode and using the algorithm for four different values of W (200, 300, 400 and 500 Mbps, respectively). It is important to mention here that in our simulator, peers do not stay as seeders in the system. They leave as soon as they finish downloading the requested files. When comparing the number of simultaneous peers using each of the approaches, we note that a lower number of simultaneous peers means that the clients are downloading faster which proves that the corresponding approach is more efficient. We notice that with very limited bandwidth budget (200 and 300 Mbps), the algorithm performs better than pure HTTP. This is due to the fact that when the seed has a very limited bandwidth budget, the share allocated to each swarm will be small. Therefore, the HTTP download time will be "high" and the overhead of switching to BitTorrent will be negligible. However, the higher the seed bandwidth gets, the bigger the overhead becomes compared to the download time in HTTP. This explains the degrade in the algorithm's performance when the seed's bandwidth budget becomes quite high (400 Mbps).

Figure 11 summarizes, for different values of W ranging from 180 Mbps to 500 Mbps, the net gain percentage and the average size of BitTorrent swarms. The first figure (Fig. 11(a)) plots the evolution of the net gain percentage with the cloud's bandwidth. Similar to the aforementioned conclusions, when the bandwidth is small (lower than 320 Mbps), the net gain percentage in download time of the clients is important (between 17.5% and 21%). However it gets lower with the increasing budget of the cloud, until reaching negative values when the seed's bandwidth is higher than 420 Mbps. This confirms our previous conclusions that the algorithm is more efficient when the cloud seed has very limited bandwidth resources. The second figure (Fig. 11(b)) presents the average number of peers in BitTorrent swarms. We notice that most of the BitTorrent swarms are very small with an average size of about 2.22 peers per swarm. This can be due to the limited sharing in UB1 system and to the fact that most of UB1 users are using the service for data backup only.

6.4. *Modified trace: bigger shared files: results*

Even though the UB1 trace has limited sharing and very small files (most of the files are smaller than 1 MB), we could achieve relatively important improvements in the system's performance. To further validate our proposal, we modified the trace in order to have bigger shared files. Our idea was to keep the same arrival pattern of the peers and just increase the size of the files that were downloaded more than once. We obtained two different modified traces:

- *trace*_1: This trace preserves the same arrival pattern as in the original trace, but we increase the size of the files smaller than 1 MB by 1 MB. For instance, if a file f_s is downloaded more than once in the original trace sample and has a file size of 100 KB, then, in *trace*_1, the same file would be 1 MB (1024 KB) bigger, that is 1124 KB. We chose this value (1 MB) because it represents the mean file size of all the files in the original trace.
- *trace*_2: This trace is obtained the same way as *trace*_1. We choose a bigger limit on size equal to 5 MB, which is the average size of a picture. This means that *trace*_2 also preserves the same arrival pattern as in the original trace, but here we increase the size of the files smaller than 5 MB by 5 MB. For instance, if a file f_s is downloaded more than once in the original trace sample and has a file size of 1 MB, in *trace*_2, the same file would be 5 MB bigger, that is 6 MB.

Clearly, when we increase the size of some files, the amount of bandwidth needed to distribute the requested file to the peers will increase too. Figure 12 presents the new required bandwidth for both traces and shows that *trace*_1 and *trace*_2 require clearly more bandwidth compared to the original requirements (Fig. 7). We apply later our algorithm on both traces and compare the results. We use the same switching constraint τ=0.4 and we fix the cloud's upload budget limit W to 400 Mbps and 1000 Mbps for *trace*_1 and *trace*_2 respectively.

Figure 13 and Table 4 summarize the results with and without the algorithm. As we can see in Fig. 13(a) and (b), the amount of bandwidth contributed by the BitTorrent clients can reach up to 100% of the cloud's initial limit. In addition, we notice important improvements in the net gain percentage that increases from about 22% for the original trace to reach over 50% when the files are bigger than 1 MB and more than 65% when the files are bigger than 5 MB. In fact, increasing the file sizes results in increased probability of switching to BitTorrent. Actually, the percentage of operations switched to BitTorrent grows from 0.76% in the original trace and reaches 19.5% in *trace*_2. This leads to a noticeable decrease in the inter-arrival time of BitTorrent swarms as seen in Fig. 13(e) and (f). The frequency of creation of new BitTorrent swarms increases from 0.21 swarms per second for the original trace to 2.59 swarms per second for *trace_1* and reaches 2.98 swarms per second for *trace_2*. Similarly, the average size of the swarms increases from 2.206 peers per swarm to reach about 4.08 peers per swarm when the sizes of the shared files become bigger.

6.5. Algorithm's performance

To measure the efficiency of the algorithm, we measure the time needed to simulate the original trace. The simulation of the whole trace (more than 225,000 operations) took around 82 min until all the downloads have finished. We also measure the time needed to calculate the bandwidth distribution for each timestamp (with the arrival of each new download operation). This time corresponds to one execution of the algorithm and we refer to it as the execution time. Figure 14(a) presents the CDF of this execution time. It varies between 0.005 and 71.566 ms. The mean and median execution times are 15.178 and 8.006 ms, respectively.

To measure the effect of the number of clients and swarms on the execution time at each round of the algorithm, we depict in Fig. 14 (b) the scatter plot of the execution time as a function of the number of clients/swarms, along with the corresponding polynomial regression of degree $d=1$. The slopes of these lines are equal to 0.016 and 0.017 for the number of clients and the number of swarms respectively.

7. Conclusions

In this paper, we propose a bandwidth allocation and protocol management algorithm that can be implemented in personal cloud systems with limited bandwidth budget. Based on the demand on the cloud and the load on each file, the cloud server is able to decide whether to use a client-server approach or a peer-assisted one to distribute that file. Our proposed algorithm for the management of cloud bandwidth achieves important improvements in terms of download time for the clients, even though in our simulator's implementation we were “stricter” on BitTorrent than HTTP. In fact, we used an estimation of the download time in HTTP that does not take into account the protocol's overheads. However, on the other hand, we added to BitTorrent the potential latency of the peers discovery phase and the delay that can be caused by pieces unavailability. Moreover, we considered the “worst case scenario” where the peers leave the system as soon as they finish download, while in reality, the synchronization process works always in the background without the user being aware of it. This means that it is more probable that the peers will stay longer, even after finishing the download and contribute more to the system. Despite that, the results prove that the use of BitTorrent in personal cloud systems can help the clients gain in download time, especially when the bandwidth resources of the seed are limited. In such conditions, the net gain percentage in the download time of all the peers exceeds 20% of their download time in most cases, based on a real trace of the UB1 system.

The original UB1 trace has limited sharing and very small files. For this reason, we modified it in order to have bigger shared files. The application of the algorithm on the modified traces results in important improvements in the download time that exceed 65% of the original download time of all the peers.

Nevertheless, several extensions can be added to the algorithm. For instance, it is possible to consider two different values of the switching constraint based on the load of the seed: $\tau_{overloaded}$ and $\tau_{not\_overloaded}$. This way, strict constraints can be put when the seed is not overloaded and loosened it up when the load on the seed increases.

Acknowledgments

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Appendix A. Inverting the gain formulas: solving the equation: $Gain(\overset{??}{w_s}, s) = \tau$

The goal of this section is to reverse the equations of the gain and get, for a given swarm s and a given file of size F_s, the amount of bandwidth needed to be provided by the seed w_s that satisfies the condition $Gain(w_s, s) \geq \tau$. We remind our reader that the gain percentage is defined as follows:

$$Gain(w_s, s) = \begin{cases} -\dfrac{\alpha_{bt}\, d_{min,s}}{F_s}, & \text{if } d_{min,s} \leq \dfrac{w_s}{L_s} \text{ and } d_{min,s} \leq \min\left\{\dfrac{w_s + \eta_s\, u_s\, L_s}{L_s}, w_s\right\} \\ 1 - \dfrac{w_s}{L_s\, d_{min,s}} - \dfrac{\alpha_{bt}\, w_s}{F_s\, L_s}, & \text{if } \dfrac{w_s}{L_s} \leq d_{min,s} \text{ and } d_{min,s} \leq \min\left\{\dfrac{w_s + \eta_s\, u_s\, L_s}{L_s}, w_s\right\} \\ 1 - \dfrac{w_s}{w_s + \eta_s\, u_s\, L_s} - \dfrac{\alpha_{bt}\, w_s}{F_s\, L_s}, & \text{if } \dfrac{w_s + \eta_s\, u_s\, L_s}{L_s} \leq \min\{d_{min,s}, w_s\} \\ 1 - \dfrac{1}{L_s} - \dfrac{\alpha_{bt}\, w_s}{F_s\, L_s}, & \text{if } w_s \leq \min\left\{d_{min,s}, \dfrac{w_s + \eta_s\, u_s\, L_s}{L_s}\right\}. \end{cases}$$

While inverting this equation and in order to be able to define correctly the interval delimiters, we need to distinguish two different cases based on the maximum of $(L_s - 1)d_{min,s}$ and $L_s\, \eta_s\, u_s$:

- *Case A*: $(L_s - 1)d_{min,s} \geq L_s\, \eta_s\, u_s$
- *Case B*: $(L_s - 1)d_{min,s} \leq L_s\, \eta_s\, u_s$

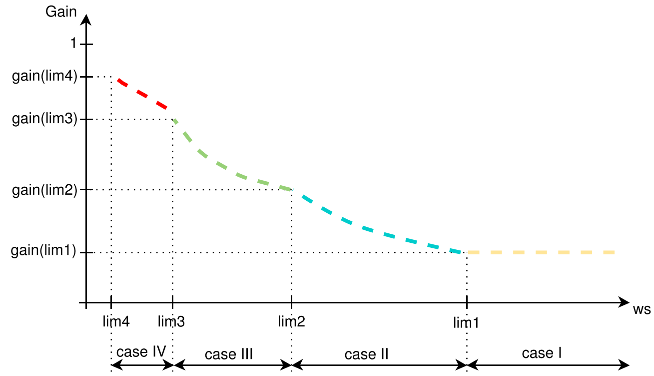

Fig. A1. General shape of the gain ratio as a function of the upload speed of the seed in case A when $(L_s-1)d_{min,s} \geq L_s\, \eta_s\, u_s$.

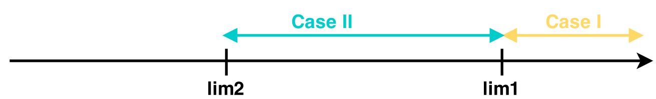

Fig. A2. Delimiter lim_2 and interval for Case II.

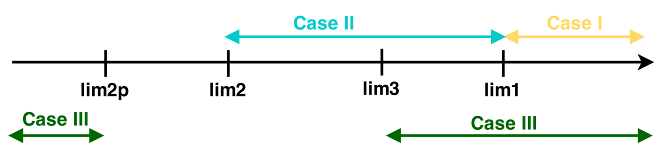

Fig. A3. Delimiter lim_{2p} and lim_3 and interval for Case III.

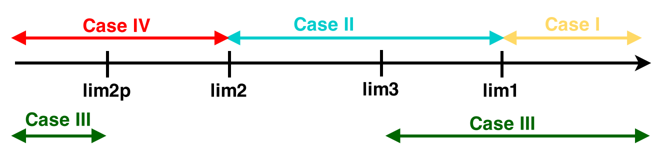

Fig. A4. Interval for Case IV.

A.1. Case A: $(L_s-1)d_{min,s} \geq L_s\, \eta_s\, u_s$

The general shape of that function is given in Fig. A1. The next step is to find expressions of the intervals delimiters (lim_1, lim_2, lim_3 and lim_4) and the corresponding gain values ($Gain(lim_1,s)$, $Gain(lim_2,s)$, $Gain(lim_3,s)$ and $Gain(lim_4,s)$).

1. lim_1 and $Gain(lim_1,s)$: The conditions of case I are the following:

$$\begin{cases} d_{min,s} \leq \dfrac{w_s}{L_s} \\ d_{min,s} \leq \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \leq w_s \end{cases} \overset{L_s \geq 1,\ \eta_s\, u_s \geq 0}{\Longrightarrow} \begin{cases} d_{min,s} \leq \dfrac{w_s}{L_s} \leq w_s \\ d_{min,s} \leq \dfrac{w_s}{L_s} \leq \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \end{cases} \Longrightarrow w_s \geq L_s\, d_{min,s} \Longrightarrow lim_1 = L_s d_{min,s}$$

The corresponding $Gain(lim_1,s)$ for case I is as follows:

$$Gain(lim_1,s) \overset{caseI}{=} Gain(L_s\, d_{min,s},s) \overset{caseI}{=} -\frac{\alpha_{bt}.d_{min,s}}{F_s}$$

• **Resolution of the equation** $Gain(\overset{??}{w_s},s)=\tau$, $\forall \tau \in [-\infty, -\frac{\alpha_{bt}\, d_{min,s}}{F_s}]$

We have $\tau \in [-\infty, -\frac{\alpha_{bt}.d_{min,s}}{F_s}]$ and $Gain(\overset{??}{w_s},s)=\tau$.

This leads to $Gain(\overset{??}{w_s},s) \in [-\infty, -\frac{\alpha_{bt}\, d_{min,s}}{F_s}]$, this means that $w_s \geq L_s\, d_{min,s}$.

Thus, $\forall \tau \in [-\infty, -\frac{\alpha_{bt}\, d_{min,s}}{F_s}]$, the optimal bandwidth that should be allocated to the swarm without violating the constraint $(Gain(w_s,s) \geq \tau)$ is $w_s = L_s.d_{min,s}$

$$\forall \tau \in \left[-\infty, -\frac{\alpha_{bt}\, d_{min,s}}{F_s}\right], \left(Gain(\overset{??}{w_s},s)=\tau\right) \Rightarrow (w_s = L_s\, d_{min,s})$$

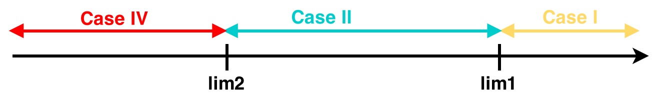

Fig. A5. Final interval to be considered for Case B.

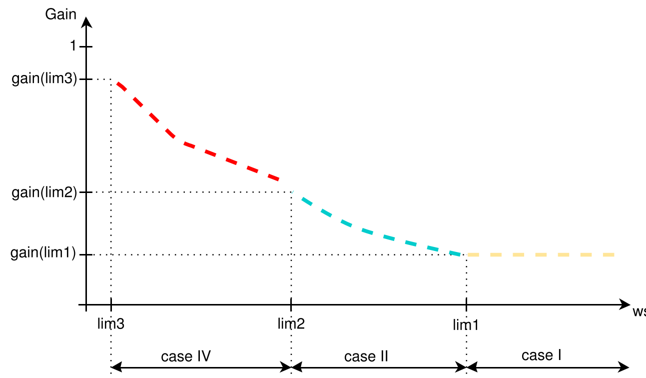

Fig. A6. General shape of the gain ratio as a function of the upload speed of the seed when $\max(d_{min,s}, L_s(d_{min,s} - \eta_s\, u_s)) = d_{min,s}$.

2. lim_2 and $Gain(lim_2, s)$: The conditions of case II are the following:

$$\begin{cases} d_{min,s} \geq \frac{w_s}{L_s}(lim_1) \\ d_{min,s} \leq \frac{w_s + \eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \leq w_s \end{cases} \Rightarrow \begin{cases} w_s \geq d_{min,s} \\ w_s \geq L_s\,(d_{min,s} - \eta_s\, u_s) \end{cases} \Rightarrow w_s \geq \max(d_{min,s}, L_s(d_{min,s} - \eta_s\, u_s))$$

$$\Rightarrow lim_2 = \max(d_{min,s}, L_s(d_{min,s} - \eta_s\, u_s)) \overset{\text{CaseA}}{\Longrightarrow} lim_2 = L_s(d_{min,s} - \eta_s\, u_s)$$

Let us verify whether $lim_1 \overset{?}{\geq} lim_2$:

$lim_1 - lim_2 = L_s\, d_{min,s} - L_s(d_{min,s} - \eta_s\, u_s) = L_s\, d_{min,s} - L_s\, d_{min,s} + L_s\, \eta_s\, u_s = L_s\, \eta_s\, u_s \geq 0$(because $L_s \geq 1,\ \eta_s \geq 0$ and $u_s \geq 0$) (verified✓)

The corresponding $Gain(lim_2, s)$ for case II is as follows:

$$Gain(lim_2, s) \overset{caseII}{=} 1 - \frac{L_s\, d_{min,s} - L_s\, \eta_s\, u_s}{L_s\, d_{min,s}} - \frac{\alpha_{bt}(L_s\, d_{min,s} - L_s\, \eta_s\, u_s)}{F_s\, L_s} = \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}$$

This formula should be verified using the gain formula for case III since the two cases share the same border lim_2:

$$Gain(lim_2, s) \overset{caseIII}{=} 1 - \frac{L_s\, d_{min,s} - L_s\, \eta_s\, u_s}{L_s\, d_{min,s} - L_s\, \eta_s\, u_s + \eta_s\, L_s\, u_s} - \frac{\alpha_{bt}\,(L_s\, d_{min,} - L_s\, \eta_s\, u_s)}{F_s\, L_s} = \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s} \text{(verified ✓)}$$

We need now to verify whether $Gain(lim_2, s) \overset{??}{\geq} Gain(lim_1, s)$

$$Gain(lim_2, s) - Gain(lim_1, s) = \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s} + \frac{\alpha_{bt}\, d_{min,s}}{F_s}\, \frac{\eta_s\, u_s}{d_{min,s}} + \frac{\alpha_{bt}\, \eta_s\, u_s}{F_s} \geq 0$$

Thus,

$$\frac{\alpha_{bt}.d_{min,s}}{F_s} \leq \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s} \quad \text{(verified ✓)}$$

- **Resolution of the equation** $Gain(\overset{??}{w_s}, s) = \tau, \forall \tau \in \left[-\frac{\alpha_{bt}.d_{min,s}}{F_s}, \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}\right]$

We have: $Gain(\overset{??}{w_s}, s) = \tau$ and $\tau \in \left[-\frac{\alpha_{bt}\, d_{min,s}}{F_s}, \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}\right]$

This means that $Gain(\overset{??}{w_s}, s) \in \left[-\frac{\alpha_{bt}\, d_{min,s}}{F_s}, \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}\right]$ which corresponds to the formula of the gain related to case II.

Let us try now to invert that formula in order to get an estimation of w_s

$$Gain(\overset{??}{w_s}, s) = \tau \overset{\text{case II}}{\Longleftrightarrow} 1 - \frac{w_s}{L_s.d_{min,s}} - \frac{\alpha_{bt}.w_s}{F_s\, L_s} = \tau \Longleftrightarrow 1 - \tau = w_s\left(\frac{1}{L_s\, d_{min,s}} + \frac{\alpha_{bt}}{F_s\, L_s}\right) \Longleftrightarrow w_s = \frac{(1-\tau)F_s\, L_s\, d_{min,s}}{F_s + d_{min,s}\, \alpha_{bt}}$$

We can then conclude that:

$$\forall \tau \in \left[-\frac{\alpha_{bt}\, d_{min,s}}{F_s}, \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s}-\eta_s\, u_s)}{F_s}\right], \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow w_s = \frac{(1-\tau)F_s\, L_s\, d_{min,s}}{F + d_{min,s}\, \alpha_{bt}}$$

3. lim_3 and $Gain(lim_3, s)$: The conditions of case III are the following:

$$\begin{cases} \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \\ \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \le w_s \end{cases} \Longrightarrow \begin{cases} w_s \le L_s(d_{min,s} - \eta_s\, u_s)(lim_2) \\ w_s \ge \dfrac{\eta_s\, L_s\, u_q}{L_s - 1} \end{cases} \Longrightarrow lim_3 = \frac{\eta_s\, L_s\, u_s}{L_s - 1}$$

Let us verify whether $lim_2 \overset{?}{\ge} lim_3$:

$$lim_2 - lim_3 = L_s(d_{min,s} - \eta_s\, u_s) - \frac{\eta_s\, L_s\, u_s}{L_s - 1} = \frac{L_s}{L_s - 1}\left(L_s(d_{min,s} - \eta_s\, u_s) - d_{min,s}\right) \ge 0 (\text{because } L_s > 1 \text{ and } d_{min,s} \le L_s(d_{min,s} - \eta_s\, u_s)) \text{ (verified ✓)}$$

The corresponding $Gain(lim_3, s)$ for case III is:

$$Gain(lim_3, s) \overset{caseIII}{=} Gain\left(\frac{\eta_s\, L_s\, u_s}{L_s - 1}, s\right) \overset{caseIII}{=} 1 - \frac{\frac{\eta_s\, L_s\, u_s}{L_s - 1}}{\frac{\eta_s\, L_s\, u_s}{L_s - 1} + \eta_s\, L_s\, u_s} - \frac{\alpha_{bt}\, \frac{\eta_s L_s\, u_s}{L_s - 1}}{F_s\, L_s} = 1 - \frac{\eta_s\, L_s\, u_s}{\eta_s\, L_s\, u_s + \eta_s\, L_s\, u_s(L_s - 1)} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s} = 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)\, F_s}$$

This expression needs to be verified also using the gain formula for case IV since the two cases share the same border lim_3:

$$Gain(lim_3, s) \overset{caseIV}{=} Gain\left(\frac{\eta_s\, L_s\, u_s}{L_s - 1}, s\right) \overset{caseIV}{=} 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\left(\frac{\eta_s\, L_s\, u_s}{L_s - 1}\right)}{F_s\, L_s} = 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s}$$

We need now to verify whether $Gain(lim_3, s) \overset{??}{\ge} Gain(lim_2, s)$

$$\begin{aligned} Gain(lim_3, s) - Gain(lim_2, s) &= 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s} - \left(\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}\right) \\ &= 1 - \frac{1}{L_s} - \frac{\eta_s\, u_s}{d_{min,s}} + \frac{\alpha_{bt}\, d_{min,s}}{F_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{F_s}\left(1 + \frac{1}{L_s - 1}\right) = \left[1 - \frac{1}{L_s} - \frac{\eta_s\, u_s}{d_{min,s}}\right] + \left[\frac{\alpha_{bt}\, d_{min,s}}{F_s} - \frac{\alpha_{bt}\, \eta_s\, u_s\, L_s}{F_s(L_s - 1)}\right] \\ &= \left(L_s(d_{min,s} - \eta_s\, u_s) - d_{min,s}\right)\left(\frac{1}{L_s\, d_{min,s}} + \frac{\alpha_{bt}}{F_s(L_s - 1)}\right) \\ &\ge 0 (\text{since all the terms are} \ge 0,\ L_s > 1 \text{ and } (d_{min,s} \le L_s(d_{min,s} - \eta_s\, u_s)) \end{aligned}$$

Thus,

$$\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s} \le 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s} \quad \text{(verified ✓)}$$

- **Resolution of the equation** $Gain(\overset{??}{w_s}, s) = \tau, \forall \tau \in \left[\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s}\right]$

We have: $Gain(\overset{??}{w_s}, s) = \tau$ and $\tau \in \left[\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s}\right]$

This means that $Gain(\overset{??}{w_s}, s) \in \left[\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, \eta_s\, u_s}{(L_s - 1)F_s}\right]$ which corresponds to the formula of the gain related to case III. Let us try now to invert that formula in order to get an estimation of w_s

$$Gain(\overset{??}{w_s}, s) = \tau \overset{\text{case III}}{\Longleftrightarrow} 1 - \frac{w_s}{w_s + \eta_s\, L_s\, u_s} - \frac{\alpha_{bt}\, w_s}{F_s\, L_s} = \tau$$

Since the resolution of this equation is too complex, we tried to simplify it by introducing the following symbols: $a = \eta_s\, L_s\, u_s$, $b = \frac{\alpha_{bt}}{F_s\, L_s}$ and $c = \tau$. The simplified equation becomes:

$$1 - \frac{w_s}{w_s + a} - b\, w_s = c$$

To solve this second degree equation, we used an online solver.[7] We obtained the following solutions:

$$w_{s_1} = \frac{\sqrt{a^2\, b^2 - 2\, a\, b\, c + 4\, a\, b + c^2} - a\, b - c}{2\, b} \quad w_{s_2} = \frac{-\left(\sqrt{a^2\, b^2 - 2\, a\, b\, c + 4\, a\, b + c^2} + a\, b + c\right)}{2\, b}$$

[7] The online solver is available at: http://www.wolframalpha.com

Clearly, $w_{s_2} < 0$ so it cannot be considered as a solution. Then, we can conclude that:

$$\forall\tau \in \left[\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}\right], \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow w_s = \frac{\sqrt{a^2\, b^2 - 2\, a\, b\, c + 4\, a\, b + c^2} - a\, b - c}{2\, b}$$

where $a = \eta_s\, L_s\, u_s$, $b = \frac{\alpha_{bt}}{F_s\, L_s}$, and $c = \tau$

4. lim_4 and $Gain(lim_4, s)$: The conditions of case IV are the following:

$$\begin{cases} w_s \le \frac{w_s + \eta_s\, L_s\, u_s}{L_s} \\ w_s \le d_{min,s} \end{cases} \Longrightarrow \begin{cases} w_s \le \frac{\eta_s\, L_s\, u}{L_s - 1}\,(lim_2) \\ w_s \le d_{min,s} \end{cases}$$

We need to compare $d_{min,s}$ and $\frac{\eta_s\, L_s\, u_s}{L_s-1}$ in order to verify lim_3.

$$d_{min,s} - \frac{\eta_s\, L_s\, u_s}{L_s - 1} = \frac{(L_s-1)d_{min,s} - \eta_s\, L_s\, u_s}{L_s - 1} = \frac{L_s(d_{min,s} - \eta_s\, u_s) - d_{min,s}}{L_s - 1} \ge 0 (\text{because } L_s > 1 \text{ and } d_{min,s} \le L_s(d_{min,s} - \eta_s\, u_s)) \quad (\text{verified } \checkmark)$$

Thus lim_3's definition is correct and there no analytic definition for lim_4. We can suppose that it can be equal to 0 since w_s can only be positive (or equal to 0). So we can suppose that $lim_4 = 0$ even thought attaining that limit means that the download might be interrupted.

We need now to calculate $\lim_{w_s \to 0} Gain(w_s, s)$ that will be considered the upper bound of the gain values

$$\lim_{w_s \to 0}(Gain(w_s, s)) \overset{caseIV}{=} \lim_{w_s \to 0}\left(1 - \frac{1}{L_s} - \frac{\alpha_{bt}.w_s}{F.L_s}\right) = 1 - \frac{1}{L_s}$$

- **Resolution of the equation** $Gain(\overset{??}{w_s}, S_f, F) = \tau$, $\forall\tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}, 1 - \frac{1}{L_s}\right]$

We have: $Gain(\overset{??}{w_s}, s) = \tau$ and $\tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}, 1 - \frac{1}{L_s}\right]$

This means that $Gain(\overset{??}{w_s}, s) \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}, 1 - \frac{1}{L_s}\right]$ which corresponds to the formula of the gain related to case IV. Let us try now to invert that formula in order to get an estimation of w_s:

$$Gain(\overset{??}{w_s}, s) = \tau \overset{\text{case IV}}{\Longleftrightarrow} 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\, w_s}{F_s\, L_s} = \tau \Longleftrightarrow \frac{\alpha_{bt}\, w_s}{F_s\, L_s} = 1 - \frac{1}{L_s} - \tau \Longleftrightarrow w_s = \frac{F_s\, L_s}{\alpha_{bt}}\left(\frac{L_s(1-\tau) - 1}{L_s}\right) \Longleftrightarrow w_s = \frac{F_s[L_s(1-\tau) - 1]}{\alpha_{bt}}$$

We can then conclude that:

$$\forall\tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta\, u}{(L_s-1)F_s}, 1 - \frac{1}{L_s}\right], \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow w_s = \frac{F_s[L_s(1-\tau) - 1]}{\alpha_{bt}}$$

General conclusion for Case A: $w_s = f(\tau)$

The equation $Gain(\overset{??}{w_s}, s) = \tau$ has the following solution:

$$w_s^{bt} = \begin{cases} L_s\, d_{min,s}, & \forall\tau \in \left[-\infty, -\frac{\alpha_{bt} d_{min,s}}{F_s}\right] \\ \frac{(1-\tau)F_s\, L_s\, d_{min,s}}{F_s + d_{min,s}\,\alpha_{bt}}, & \forall\tau \in \left[-\frac{\alpha_{bt}\, d_{min,s}}{F_s}, \frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}\right] \\ \frac{\sqrt{a^2b^2 - 2abc + 4ab + c^2} - ab - c}{2\, b}, & \forall\tau \in \left[\frac{\eta_s\, u_s}{d_{min,s}} - \frac{\alpha_{bt}(d_{min,s} - \eta_s\, u_s)}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}\right] \\ \frac{F_s[L_s(1-\tau) - 1]}{\alpha_{bt}}, & \forall\tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\,\eta_s\, u_s}{(L_s-1)F_s}, 1 - \frac{1}{L_s}\right] \\ \nexists, & \forall\tau \in \left[1 - \frac{1}{L_s}, +\infty\right] \end{cases}$$

Where:

$a = \eta_s\, L_s\, u_s, b = \frac{\alpha_{bt}}{F_s\, L_s}$ and $c = \tau$

A.2. Case B: $(L_s - 1)d_{min,s} \le L_s\,\eta_s\, u_s$

In this case, the intervals and the delimiters are not as evident as in Case A. So let us start first by studying the limits in the gain cases and try to locate the cases based on their order.

A.2.1. Fixing the interval delimiters

For each interval we part from its constraints and determine the range of values pf w_s in each of these intervals. The goal is to verify if these intervals are disjoint and do not superimpose (Figs. A2 and A3).

1. lim_1: (**case I**)

$$\begin{cases} d_{min,s} \le \dfrac{w_s}{L_s} \\ d_{min,s} \le \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \le w_s \end{cases} \Rightarrow d_{min,s} \le \frac{w_s}{L_s} \Rightarrow w_s \ge lim_1 = L_s\, d_{min,s}$$

2. lim_2: (**case II**)

$$\begin{cases} d_{min,s} \ge \dfrac{w_s}{L_s} \\ d_{min,s} \le \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \le w_s \end{cases} \Longrightarrow \begin{cases} w_s \le L_s\, d_{min,s} = lim_1 \\ w_s \ge d_{min,s} \\ w_s \ge L_s(d_{min,s} - \eta_s\, u_s) \end{cases} \overset{\text{Case B}}{\Longrightarrow} \begin{cases} w_s \le lim_1 \\ w_s \ge d_{min,s} = lim_2 \end{cases}$$

Comparing lim_1 and lim_2: We know that $L_s > 1$ and $d_{min,s} > 0$ Thus $L_s\, d_{min,s} > d_{min,s} \Rightarrow lim_1 > lim_2$

3. lim_3 and $\lim_{2p}$(**case III**)

$$\begin{cases} \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \\ \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \le w_s \end{cases} \Longrightarrow \begin{cases} w_s \le L_s(d_{min,s} - \eta_s\, u_s) = lim_{2p} \\ w_s \ge \dfrac{\eta\, L_s\, u_s}{L_s - 1} = lim_3 \end{cases}$$

Comparing lim_{2p} and lim_2: Based on the conditions of Case B, we already know that $lim_{2p} \le lim_2$

Comparing lim_3 and lim_2:

$$lim_3 - lim_2 = \frac{\eta_s\, L_s\, u_s}{L_s - 1} - d_{min,s} = \frac{\eta_s\, L_s\, u_s - (L_s - 1) d_{min,s}}{L_s - 1} = \frac{1}{L_s - 1}(d_{min,s} - L_s(d_{min,s} - \eta_s\, u_s))$$
$$\ge 0 (\text{because } L_s > 1 \text{ and } d_{min,s} \ge L_s(d_{min,s} - \eta_s\, u_s)) \Rightarrow lim_3 \ge lim_2$$

Comparing lim_3 and lim_1:

$$lim_3 - lim_1 = \frac{\eta_s\, L_s\, u_s}{L_s - 1} - L_s\, d_{min,s} = \frac{\eta_s\, L_s\, u_s - L_s(L_s - 1).d_{min,s}}{L_s - 1} \frac{\left(L_s(d_{min,s} - \eta_s\, u_s) - L_s^2\, d_{min,s}\right)}{L_s - 1}$$

We have $L_s \ge 1 \Rightarrow L_s^2 \ge 1$ & $d_{min,s} \ge 0 \Rightarrow L_s^2 . d_{min,s} \ge d_{min,s}$

and since $\begin{cases} d_{min,s} \ge L_s(d_{min,s} - \eta_s\, u_s) \\ L_s^2\, d_{min,s} \ge d_{min,s} \end{cases} \Rightarrow L_s^2\, d_{min,s} \ge L_s(d_{min,s} - \eta_s\, u_s)$

Thus $lim_3 - lim_1 \le 0 \Rightarrow lim_1 \ge lim_3$

4. lim_4: (**case IV**)

$$\begin{cases} w_s \le \dfrac{w_s + \eta_s\, L_s\, u_s}{L_s} \\ w_s \le d_{min,s} \end{cases} \Rightarrow \begin{cases} w_s \le \dfrac{\eta_s\, L_s\, u_s}{L_s - 1} \\ w_s \le d_{min,s} \end{cases} \Rightarrow \begin{cases} w_s \le lim_3 \\ w_s \le lim_2 \end{cases} \overset{lim_3 \ge lim_2}{\Longrightarrow} w_s \le lim_2$$

A.2.2. Interpretation of the superimposed cases

Based on Fig. A4, we can distinguish 3 intervals where there are superimposed cases which are:

- **Interval 1**. $[-\infty, lim_{2p}] = [-\infty, L_s(d_{min,s} - \eta_s\, u_s)]$: superimposition of case IV and case III
- **Interval 2**. $[lim_3, lim_1] = \left[\frac{\eta_s\, L_s\, u_s}{L_s - 1}, L_s\, d_{min,s}\right]$: superimposition of case II and case III
- **Interval 3**. $[lim_1, +\infty[= [L_s\, d_{min,s}, +\infty]$: superimposition of case I and case III

Let us check interval by interval the implications of such a superimposition. For each interval we will define the new constraints resulting from the intersection of the corresponding cases and define accordingly the gain expression. The goal is to demonstrate that the solution will be the same for both cases.

i. **Interval 1**:

$$\begin{cases} \text{Case IV}: \begin{cases} w_s \le d_{min,s} \\ w_s \le \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \end{cases} \\ \text{Case III}: \begin{cases} \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \\ \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le w_s \end{cases} \end{cases} \Rightarrow \quad w_s = \frac{w_s+\eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \Rightarrow w_s = \frac{L_s\, \eta_s\, u_s}{L_s-1}$$

Verification of the gain expression in both cases: Let us now verify that $Gain(w_s, s)$ has the same expression in both cases IV and III when $w_s = \frac{\eta_s\, L_s\, u_s}{L_s-1}$.

$$Gain\left(\frac{L_s\, \eta_s\, u_s}{L_s-1}, s\right) \overset{\text{case IV}}{=} 1-\frac{1}{L_s}-\frac{\alpha_{bt}\dfrac{\eta_s\, L_s\, u_s}{L_s-1}}{F_s\, L_s} = 1-\frac{1}{L_s}-\frac{\alpha_{bt}\, \eta_s\, u_s}{F_s\,(L_s-1)}$$

$$Gain\left(\frac{\eta_s\, L_s\, u_s}{L_s-1}, s\right) \overset{\text{case III}}{=} 1-\frac{\dfrac{\eta_s\, L_s\, u_s}{L_s-1}}{\dfrac{\eta_s\, L_s\, u_s}{L_s-1}+\eta_s\, L_s\, u_s}-\frac{\alpha_{bt}.\dfrac{\eta_s\, L_s\, u_s}{L_s-1}}{F_s\, L_s} = 1-\frac{1}{1+(L_s-1)}-\frac{\alpha_{bt}.\eta_s\, u_s}{F_s\,(L_s-1)} = 1-\frac{1}{L_s}-\frac{\alpha_{bt}.\eta_s\, u_s}{F_s\,(L_s-1)}$$

For this interval, both cases have the same expression. Thus, we can just consider just one of the cases instead of working with both. The best choice seems to be *case* IV since it has a simpler formulas.

ii. **Interval 2**:

$$\begin{cases} \text{Case II}: \begin{cases} d_{min,s} \ge \dfrac{w_s}{L_s} \\ d_{min,s} \le \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \le w_s \end{cases} \\ \text{Case III}: \begin{cases} \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \\ \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le w_s \end{cases} \end{cases} \Rightarrow \frac{w_s}{L_s} \le d_{min,s} = \frac{w_s+\eta_s\, L_s\, u_s}{L_s} \le w_s$$

$$\Rightarrow w_s = L_s\big(d_{min,s}-\eta_s\, u_s\big)$$

Verification of the gain expression in both cases: Let us now verify that $Gain(w_s, s)$ has the same expression in both cases II and III when $w_s = L_s(d_{min,s}-\eta_s\, u_s)$:

$$Gain(L_s\, d_{min,s}, s) \overset{\text{case II}}{=} 1-\frac{L_s(d_{min,s}-\eta_s\, u_s)}{L_s d_{min,s}}-\frac{\alpha_{bt}\, L_s(d_{min,s}-\eta_s\, u_s)}{F_s\, L_s}$$

$$Gain(L_s\, d_{min,s}, s) \overset{\text{case III}}{=} 1-\frac{L_s(d_{min,s}-\eta_s\, u_s)}{L_s(d_{min,s}-\eta_s\, u_s)+\eta_s\, L_s\, u_s}-\frac{\alpha_{bt}\, L_s(d_{min,s}-\eta_s\, u_s)}{F_s\, L_s} = 1-\frac{L_s(d_{min,s}-\eta_s\, u_s)}{L_s\, d_{min,s}}-\frac{\alpha_{bt}\, L_s(d_{min,s}-\eta_s\, u_s)}{F_s\, L_s}$$

For this interval, both cases have the same expression. Thus, we can just consider just one of the cases instead of working with both. The best choice seems to be *case* II since it has simpler formulas.

iii. **Interval 3**: *Constraints definition*:

$$\begin{cases} \text{Case I}: \begin{cases} d_{min,s} \le \dfrac{w_s}{L_s} \\ d_{min,s} \le \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \\ d_{min,s} \le w_s \end{cases} \\ \text{Case III}: \begin{cases} \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le d_{min,s} \\ \dfrac{w_s+\eta_s\, L_s\, u_s}{L_s} \le w_s \end{cases} \end{cases} \overset{L_s>1}{\Longrightarrow} \quad d_{min,s} = \frac{w_s+\eta_s\, L_s\, u_s}{L_s} \le \frac{w_s}{L_s}$$

$$\overset{\frac{L_s \ge 1}{\eta_s\, u_s \ge 0}}{\Longrightarrow} \quad \begin{cases} d_{min,s} = \frac{w_s+\eta_s\, L_s\, u_s}{L_s} \\ \eta_s\, L_s\, u_s = 0 \end{cases} \Rightarrow w_s = L_s\, d_{min,s}$$

Verification of the gain expression in both cases: Let us now verify that $Gain(w_s, s)$ has the same expression in both cases II and III when $w_s = L_s\ d_{min,s}$:

$$Gain(L_s.d_{min,s}, s) \overset{\text{case III}}{=} 1 - \frac{L_s\ d_{min,s}}{L_s\ d_{min,s} + \eta_s\ L_s\ u_s} - \frac{\alpha_{bt}\ L_s\ d_{min,s}}{F_s\ L_s} \overset{\eta_s\ u_s = 0}{=} 1 - 1 - \frac{\alpha_{bt}\ d_{min,s}}{F_s} = -\frac{\alpha_{bt}\ d_{min,s}}{F_s} \overset{\text{case I}}{=} Gain(L_s\ d_{min,s}, s)$$

For this interval, both cases have the same expression. Thus, we can just consider just one of the cases instead of working with both. The best choice seems to be **case I** since it has a simpler formulas.

After the verification of the superimposed intervals, we can just consider the following intervals (Fig. A5).

A.2.3. Inverting the gain

The goal of this section is to derive a potential equation of the gain ratio as a function of the seed's upload speed. The general shape of that function is given in Fig. A1. We have already defined the intervals delimiters (lim_1 and lim_2) and lim_3 is the lower bound of w_s (that is 0). We just need to verify the corresponding gain values ($Gain(lim_1, s), Gain(lim_2, s)$ and $Gain(lim_3, s)$) that should be equal to the ones already defined in Case A (Fig. A6).

1. lim_1 and $Gain(lim_1, s)$: We have already found that $lim_1 = L_s\ d_{min,s}$. The corresponding gain has been already calculated and verified for both case I and case II. It is equal to: $Gain(lim_1, s) = -\frac{\alpha_{bt}\ d_{min,s}}{F_s}$.

   • **Resolution of the equation** $Gain(\overset{??}{w_s}, s) = \tau, \forall \tau \in]-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F_s}]$

   Since $\tau \in]-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F_s}]$ and $Gain(\overset{??}{w_s}, s) = \tau$.

   This leads to $Gain(\overset{??}{w_s}, s) \in \left[-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F_s}\right]$, this means that $w_s \geq L_s\ d_{min,s}$.

   Thus, $\forall \tau \in \left[-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F_s}\right]$, the optimal bandwidth that should be allocated to the swarm without violating the constraint ($Gain(w_s, s) \geq \tau$) is $w_s = L_s\ d_{min,s}$

   $$\forall \tau \in \left[-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F}\right], \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow (w_s = L_s\ d_{min,s})$$

2. lim_2 and $Gain(lim_2, s)$: We have already found that $lim_2 = d_{min,s}$. Now, we need to calculate $Gain(d_{min,s}, s)$ for both cases II and IV and we should obtain the same value to which we will refer to as: $Gain(lim_2, s)$

   $$Gain(d_{min,s}, s) \overset{\text{case II}}{=} 1 - \frac{d_{min,s}}{L_s\ d_{min,s}} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s} = 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s} \overset{\text{case IV}}{=} Gain(d_{min,s}, s)$$

   We obtain finally that: $Gain(lim_2, s) = 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}$.

   • **Resolution of the equation** $Gain(\overset{??}{w_s}, s) = \tau, \forall \tau \in \left[-\frac{\alpha_{bt}\ d_{min,s}}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}\right]$

   Since $\tau \in \left[-\frac{\alpha_{bt}\ d_{min,s}}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}\right]$ and $Gain(\overset{??}{w_s}, s) = \tau$.

   This leads to $Gain(\overset{??}{w_s}, s) \in \left[-\frac{\alpha_{bt}\ d_{min,s}}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}\right]$, which corresponds to the formula of the gain related to case II.

   Let us try now to invert that formula in order to get an estimation of w_s

   $$Gain(\overset{??}{w_s}, s) = \tau \overset{\text{case II}}{\Longleftrightarrow} 1 - \frac{w_s}{L_s\ d_{min,s}} - \frac{\alpha_{bt}\ w_s}{F_s\ L_s} = \tau \Longleftrightarrow 1 - \tau = w_s\left(\frac{1}{L_s\ d_{min,s}} + \frac{\alpha_{bt}}{F_s\ L_s}\right) \Longleftrightarrow 1 - \tau = w_s\left(\frac{F_s + d_{min,s}\ \alpha_{bt}}{F_s\ L_s\ d_{min,s}}\right) \Longleftrightarrow w_s = \frac{(1-\tau)F_s\ L_s\ d_{min,s}}{F_s + d_{min,s}\ \alpha_{bt}}$$

   We can then conclude that:

   $$\forall \tau \in \left[-\frac{\alpha_{bt}\ d_{min,s}}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}\right], \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow w_s = \frac{(1-\tau)F_s\ L_s\ d_{min,s}}{F_s + d_{min,s}\ \alpha_{bt}}$$

3. lim_3 and $Gain(lim_3, s)$: the definition of these parameters is very similar to the one done for lim_4 and $Gain(lim_4, s)$ in Case A.
   Since w_s can only be positive (or equal to 0), we can suppose that $lim_3 = 0$ even thought attaining that limit means that the download might be interrupted.
   We need now to calculate $\lim_{w_s \to lim_3} Gain(w_s, s)$ that will be considered as the upper bound of the gain values:

   $$\lim_{w_s \to lim_3} (Gain(w_s, s)) \overset{caseIV}{=} \lim_{w_s \to 0}\left(1 - \frac{1}{L_s} - \frac{\alpha_{bt}.w_s}{F_s\ L_s}\right) = 1 - \frac{1}{L_s} = Gain(lim_3, s)$$

   • **Resolution of the equation** $Gain(\overset{??}{w_s}, s) = \tau, \forall \tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}, 1 - \frac{1}{L_s}\right]$

   We have: $Gain(\overset{??}{w_s}, s) = \tau$ and $\tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}, 1 - \frac{1}{L_s}\right]$

   This means that $Gain(\overset{??}{w_s}, s) \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}, 1 - \frac{1}{L_s}\right]$ which corresponds to the formula of the gain related to case IV.

Let's try now to invert that formula in order to get an estimation of w_s

$$Gain(\overset{??}{w_s}, s) = \tau \overset{\text{case IV}}{\Longleftrightarrow} 1 - \frac{1}{L} - \frac{\alpha_{bt}\ w_s}{F_s\ L_s} = \tau \Longleftrightarrow \frac{\alpha_{bt}\ w_s}{F_s\ L_s} = 1 - \frac{1}{L_s} - \tau \Longleftrightarrow w_s = \frac{F_s\ L_s}{\alpha_{bt}}\left(\frac{L_s(1-\tau)-1}{L_s}\right) \Longleftrightarrow w_s = \frac{F_s[L_s(1-\tau)-1]}{\alpha_{bt}}$$

We can then conclude that:

$$\forall \tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}, 1 - \frac{1}{L_s}\right[, \left(Gain(\overset{??}{w_s}, s) = \tau\right) \Rightarrow w_s = \frac{F_s(L_s\ (1-\tau)-1)}{\alpha_{bt}} \tag{A.1}$$

General conclusion for Case B: $w_s = f(\tau)$

The equation: $Gain(\overset{??}{w_s}, s) = \tau$ has the following solution:

$$w_s^{bt} = \begin{cases} L_s\ d_{min,s}, & \forall \tau \in \left[-\infty, -\frac{\alpha_{bt}\ d_{min,s}}{F_s}\right] \\ \frac{(1-\tau)F_s\ L_s.d_{min,s}}{F_s + d_{min,s}\ \alpha_{bt}}, & \forall \tau \in \left[-\frac{\alpha_{bt}\ d_{min,s}}{F_s}, 1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}\right] \\ \frac{F_s[L_s(1-\tau)-1]}{\alpha_{bt}}, & \forall \tau \in \left[1 - \frac{1}{L_s} - \frac{\alpha_{bt}\ d_{min,s}}{F_s\ L_s}, 1 - \frac{1}{L_s}\right] \\ \nexists, & \forall \tau \in \left[1 - \frac{1}{L_s}, +\infty\right] \end{cases}$$

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